THE END POINT COMPACTIFICATION OF MANIFOLDS

FRANK RAYMOND

Introduction. This paper originated in trying to show that the one point compactification of an orientable generalized n-manifold (n-gm) with cohomology isomorphic to Euclidean n-space was an orientable n-gm. Heretofore, in papers on transformation groups where this was relevant, it was stated as an extra assumption.[1] The solution to this problem is given as a corollary to the main theorems which characterize the orientable (or locally orientable) generalized manifolds whose Freudenthal end point compactification is again an orientable (or locally orientable) generalized n-manifold (see 4.5 and 4.13). In the first section we give a new characterization of the Freudenthal end point compactification in terms of inverse limits. We show as in Specker [**10**] that a certain 0-dimensional cohomology group measures the extent of this compactification.

Higher dimensional analogues of this cohomology group have been used by Conner [**4**] in proving, e.g., that a simply connected locally Euclidean n-manifold whose 1 point compactification is a locally Euclidean n-manifold cannot be fibered by a non-trivial compact fiber. He has called these groups the cohomology of the ideal boundary. These groups are further studied and the homology analogue is derived (see § 2). The main Lemma (2.16) is an exact sequence which relates the homology of the ideal boundary with the homology of a given compactification and the local homology groups at infinity. This exact sequence together with our characterization of the Freudenthal compactification gives the main theorems.

Applications are given in § 3 to Poincaré duality, in § 5 to open 2-manifolds, and in § 6 special mappings of manifolds.

Throughout this paper X will denote a locally campct, locally connected, connected Hausdorff space. If S is a locally compact Hausdorff space, $A(S)$ will denote the collection of open subsets of S whose closure is compact. When the generic space is not necessarily locally connected, as is usually the case in § 2 and part of § 4, it will be denoted by the letter S.

1. The Freudenthal end point compacification.

1.1. LEMMA. *If $V, U \in A(X)$ such that $\overline{V} \subset U$, then at most a finite number of components of $X - \overline{V}$ meet in $X - \overline{U}$. In particular,*

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[1] See, for example, Montgomery and Mostow, Toroid transformation groups on Euclidean space, Ill. J. of Math. **2** (1958), or [**14**].