SEQUEL TO A PAPER OF A. E. TAYLOR

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Introduction. In § 7 of the paper of the title [3], certain questions are posed. The theorem presented in § 1 of this sequel answers these questions. The presentation in § 1 proceeds independently of [3] and is self-contained.

The notions of Mittag-Leffler development and finite type, as introduced in [3], call for some comment; in fact the notion of finite type is not made entirely clear in [3]. These and related matters are taken up in §§ 2 and 3 below.

In this sequel to Taylor's paper, all convergence of operators is with respect to the uniform operator topology.

1. THEOREM. *Let X be a complex Banach space, and $E_1, E_2, \cdots$ an infinite sequence of bounded non-zero projections of X into itself. Suppose $\{\lambda_n\}$ is a sequence of complex numbers such that the series*

$$\sum_{n=1}^{\infty} \lambda_n E_n \tag{1.1}$$

converges to an operator B. Then

$$\lambda_n \to 0 \,. \tag{1.2}$$

If, in addition to the above hypotheses, $E_m E_n = 0$ for $m \neq n$, and the λ_n's are distinct and non-zero, then:

(1.3) *The spectrum of B, $\sigma(B)$, is the set of points $\{0, \lambda_1, \cdots, \lambda_n, \cdots\}$. In view of (1.2), 0 is the sole accumulation point of $\sigma(B)$.*

(1.4) *The resolvent of B, $R_\lambda(B)$, is given by*

$$R_\lambda(B) = \frac{I}{\lambda} + \sum_{n=1}^{\infty} \frac{\lambda_n}{\lambda(\lambda - \lambda_n)} E_n \,,$$

where I is the identity operator and the series converges uniformly with respect to λ on each compact subset of the resolvent set, $\rho(B)$.

(1.5) *Each of the points λ_n is a simple pole of $R_\lambda(B)$, and the residue of $R_\lambda(B)$ at λ_n is E_n.*

Proof. For each n the idempotence of E_n implies that $\|E_n\| \leq \|E_n\|^2$, whence, since $E_n \neq 0$, $\|E_n\| \geq 1$. Hence

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