# THE GIBBS PHENOMENON FOR HAUSDORFF MEANS

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The existence of a Gibbs phenomenon for the Hausdorff summability method given by $dg(x)[dg$ any measure on $[0, 1]$ with total mass 1 and no mass point at 0] is equivalent to the statement

$$\int_0^1 \int_0^{Ax} \frac{\sin t}{t} dt\, dg(x) > \frac{\pi}{2} \text{ for some } A > 0. \qquad \text{(See [3])}$$

Recently A. Livingston [1] has treated the case of a $dg$ composed of finitely many mass points and has shown that the Gibbs phenomenon holds under certain additional restrictions. Our result, which follows, contains his and does not require these additional restrictions.

THEOREM 1. *Let $dg$ have at least 1 mass point and satisfy $\int_0^1 (|\, dg(x)\,|/x^2) < \infty$ then the Gibbs phenomenon occurs for $dg$.*

[In particular if $dg$ consists of finitely many mass points, then we have the Gibbs phenomenon].

It seems peculiar that any condition at 0 is necessary, and L. Lorch had even made the conjecture that the Gibbs phenomenon persists for any $dg$ with unbounded Lebesgue constants [and so certainly for any $dg$ with at least one mass point]. Nevertheless we show that some condition at 0 is necessary.

THEOREM 2. *There exists a positive $dg$ composed solely of mass points for which the Gibbs phenomenon does not hold.*

Thus Lorch's conjecture is definitely false and although our Theorem 1 is by no means best possible it is qualitatively the correct one.

*Proof of Theorem* 1. We are required to prove that, for a $dg$ satisfying the hypotheses, there is an $A > 0$ for which

$$F(A) = \int_0^1 \int_{Ax}^{\infty} \frac{\sin t}{t} dt \quad dg(x) \text{ becomes negative.}$$

This we accomplish by showing that

1. $\displaystyle\int_1^y F(A)dA$ remains bounded as $y \to \infty$

2. $F(A) \notin L^1(1, \infty)$.

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