REPRODUCING KERNELS AND ORTHOGONAL KERNELS FOR ANALYTIC DIFFERENTIALS ON RIEMANN SURFACES

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Introduction. In the case of plane regions of finite connectivity, there exist (Bergman [2]) some kernel functions which possess reproducing or orthogonal properties with respect to the space of analytic exact differentials. Such Bergman kernels can be defined in a constructive way, and are related to some derivatives of the Neumann function for the region. Starting with the Green's function, one may construct the corresponding Bergman kernels for the space of analytic differentials in the region.

On the other hand, by Hilbert space methods one may prove the existence of corresponding kernels on arbitrary Riemann surfaces (Ahlfors and Sario [1]) = these methods are of the nonconstructive type.

In this paper we shall construct actually such kernels for the space of analytic differentials on an arbitrary Riemann surface W. We shall first establish the relationship between the kernels and some principal functions in the case of a compact bordered $\bar{W}$. The principal functions solve some well defined boundary value problems on $\bar{W}$. By a canonical exhaustion by regular regions (with compact bordered closures) the results are extended to an open Riemann surface W.

Chapter I is preliminary in nature and contains important theorems as well as definitions used in the sequel. It is a brief survey of the theory of principal functions and of the theory of differentials on Riemann surfaces (Ahlfors and Sario [1]).

In Chapter II, we introduce the concept of abstract reproducing and orthogonal kernels (Bergman [2], Schiffer [5]). We prove a uniqueness theorem for the reproducing kernel corresponding to a closed subspace Γ_α of Γ_a, the space of analytic differentials on a Riemann surface W, and for the related orthogonal kernels with a given analytic singularity. We then determine some functionals which are extremalized by reproducing and orthogonal kernels. In particular, the reproducing kernel $k_0(z, \zeta)dz$ for Γ_α minimizes the expression

$$\| adz \|^2 - 2Rea(\zeta)$$

among all differentials $adz \in \Gamma_\alpha$, and the kernel $h_0(z, \zeta)dz$ with singularity

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