ON THE GREEN'S FUNCTION OF AN N-POINT BOUNDARY VALUE PROBLEM

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1. Introduction. In a recent paper [3], D. V. V. Wend made use of the Green's functions $g_2(x, s)$, $g_3(x, s)$ for the boundary value problems

$$u'' = 0;\ u(a_1) = u(a_2) = 0, \qquad (a_1 < a_2)\ ,$$
$$u''' = 0;\ u(a_1) = u(a_2) = u(a_3) = 0, \quad (a_1 < a_2 < a_3)\ .$$

In particular, he showed that if $a_1 \geqq 0$, then

$$|g_2(x, s)| < a_2\ , \quad |g_3(x, s)| < a_3^2$$

for $a_1 \leqq x, s \leqq a_2$ or $a_1 \leqq x, s \leqq a_3$ respectively. He conjectured that if $g_n(x, s)$ is the Green's function for the boundary value problem

$$u^{(n)} = 0;\ u(a_1) = \cdots = u(a_n) = 0,\ (a_1 < a_2 < \cdots < a_n)\ , \tag{1.1}$$

then

$$|g_n(x, s)| < a_n^{n-1}, \quad a_1 \leqq x, s \leqq a_n\ ,$$

(if $a_1 \geqq 0$) and states in a footnote that this conjecture has been verified for $n < 6$. Assuming this conjecture valid he uses the inequality to obtain a lower bound for the mth positive zero of a solution of the differential equation

$$y^{(n)} + f(x)y = 0 \tag{1.2}$$

where $f(x)$ is continuous and complex-valued on $0 \leqq x < \infty$. In this proof, all zeros of the solution are *counted* as though they were *simple* zeros.

In this paper, we consider a more general boundary value problem allowing for multiple zeros of $y(x)$. Let $g_n(x, s)$ now denote the Green's function of the differential system

$$\begin{cases} y^{(n)} = 0\ , \\ y(a_i) = y'(a_i) = y''(a_i) = \cdots = y^{(k_i)}(a_i) = 0\ . \qquad 1 \leqq i \leqq r\ , \end{cases} \tag{1.3}$$

where $a_1 < a_2 < \cdots < a_r$, $0 \leqq k_i$, $k_1 + k_2 + \cdots + k_r + r = n$. In §2, we shall prove that

$$|g_n(x, s)| \leqq \frac{\prod_{i=1}^{r} |x - a_i|^{k_i+1}}{(n-1)!\,(a_r - a_1)} \leqq \left(\frac{n-1}{n}\right)^{n-1} \frac{(a_r - a_1)^{n-1}}{n!} \tag{1.4}$$

for $a_1 < x, s < a_r$. In the case $r = n$, Wend's conjecture is thus verified,

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