DIMENSIONAL INVERTIBILITY

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We report here upon another aspect of our continuing investigation of invertibility (see [5, 6]) and its applications in the theory of manifolds.

All spaces considered here are separable and metric.

A separable metric space X will be said to be *k-invertible*, $0 \leqq k \leqq \dim X$, if for each nonempty open set U and each compact proper subset C of dimension $\leqq k$, there is a homeomorphism h of X onto itself such that $h(C)$ lies in U. Then we say that X is *strongly k-invertible* if for each nonempty open set U and each *closed* proper subset C of dimension $\leqq k$, there is a homeomorphism h of X onto itself such that $h(C)$ lies in U.

Clearly, "strongly k-invertible" implies "k-invertible" and the two properties coincide in compact spaces. If $\dim X = n$, then "invertible" and "strongly n-invertible" are equivalent but, for instance, E^n is n-invertible and not invertible. We remark that k-invertibility is a strong form of near-homogeneity and says that compact k-dimensional subsets are "small under homeomorphisms." In the case of an n-manifold, k-invertibility is equivalent to the condition that every compact set of dimension k lie in an open n-cell.

We first collect some results on 0-invertible spaces, most of these results being simple generalizations of theorems to be found in [5]. The first of these requires no proof here.

THEOREM 1. *The orbit of any point in a* 0*-invertible space is dense in the space.*

THEOREM 2. *Each orbit in a* 0*-invertible space is itself* 0*-invertible.*

Proof. Let 0 be the orbit of any point in a 0-invertible space X. Let U be an open subset of 0 and C be a compact 0-dimensional proper subset of 0. Then there is an open set V in X such that $V \cap 0 = U$ and, by 0-invertibility, there is a space homeomorphism h such that $h(C)$ lies in V. But by definition of 0 as an orbit, $h(C)$ also lies in 0, hence $h(C)$ lies in $V \cap 0 = U$.

COROLLARY. *Each* 0*-invertible space is a union of disjoint, dense homogeneous,* 0*-invertible subspaces.*

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