CENTERS OF PURITY IN ABELIAN GROUPS

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This note is a supplement to the paper [5][1] of J. D. Reid "On subgroups of an abelian group maximal disjoint from a given subgroup." Our main result is based on the observation that in the case of primary groups, a bit of extra information can be gleaned from Reid's Theorem 2.1. We are led to the following characterization of the "centers of purity" in a p-group.

THEOREM 1. *Let G be a p-group. For each integer $k \geqq 0$, define $P_k = G[p] \cap p^k G$. Let $P_\infty = G[p] \cap G^1$, and $P_{\infty+1} = P_{\infty+2} = 0$. Let H be a subgroup of G. Then H is a center of purity in G (that is, every subgroup of G which is maximal with respect to disjointness from H is pure) if and only if there exists k with $0 \leqq k \leqq \infty$ such that*

$$P_k \supseteqq H[p] \supseteqq P_{k+2} .$$

It is easy to see that if G is a torsion group and H is a subgroup of G, then H is a center of purity in G if and only if every p-component H_p of H is a center of purity in the corresponding p-component G_p of G. Thus, Theorem 1 can be used to determine the centers of purity in torsion groups. The following result shows that the centers of purity in arbitrary groups can also be characterized.

THEOREM 2. *A subgroup H of an abelian group G is a center of purity in G if and only if the following two conditions are satisfied:*

(i) *the torsion subgroup H_t of H is a center of purity in the torsion subgroup G_t of G;*

(ii) *either G/H is a torsion group, or else, for all primes p,*

$$H[p] \subseteqq \bigcap_{n=0}^{\infty} p^n G .$$

The problem of characterizing centers of purity in p-groups was first posed by J. M. Irwin in [2]. Irwin showed that any subgroup of a p-group G which is maximal disjoint from G^1 is pure in G. In [3], Irwin and Walker extended this result to arbitrary abelian groups. They also showed that if G is a torsion group and H is a subgroup

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