# CHAINS AND GRAPHS OF OSTROM PLANES

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1. In 1961, in a letter to D. Hughes, T. G. Ostrom communicated a process that, as developed by Hughes and set forth by A. A. Albert [1], transformed a projective plane of a particular type into another using a coordinatizing ring of the first as a tool. This process may be modified to make more direct use of the algebra to a point where, indeed, it may be employed to create new rings out of old without the mediation of a plane. On the other hand the process may be dualized to alleviate a disadvantage of the essentially involutory nature of the original; from a given initial plane the Ostrom process gives one new plane; if repeated the original plane results. In the process to be discussed below a number of planes result, and, in particular, from a Desarguesian plane of order at least 9, three others, the Hall plane, its dual, and a self-dual plane make a complete set. Recently, Ostrom has published in [5] a development of his original process. We shall refer primarily to [1] as it more directly affects the development of the results to be presented.

2. First we establish some notation: Let $\pi$ be a finite projective plane coordinatized by a ternary ring $R$ whose additive structure is a group. For purposes of symmetry, we modify the usual notation and denote by $y = x \cdot m \circ b$ the line through the point $(m)$ of $L_\infty$ and the point $(0, -b)$. Let $\pi^*$ be the dual of $\pi$ and let it be coordinatized by $R^*$ where $R^*$ is defined by $b = m \cdot x \circ y$ in $R^*$ when $y = x \cdot m \circ b$ in $R$. We note that, if $\pi$ (and $R$) are such that $x \cdot m \circ b = xm - b$ for all $x, m, b$, then $R^*$ is just the multiplicative mirror image of $R$ and if, further, $R$ is commutative, $R = R^*$.

Second we assume some additional restrictions on $R$ (and thereby on $R^*$, $\pi$, and $\pi^*$).

(a) The additive structure of $R$ is that of an abelian group.

(b) $R$ is a vector space of dimension 2 over a field $K$ whose elements commute with all elements of $R$ in the standard binary multiplication in $R$.

(c) For $a, b \in R$, $\alpha, \delta \in K$,

$$\alpha(\delta a) = (\alpha\delta)a$$
$$(\alpha + \delta)a = \alpha a + \delta a$$
$$\alpha(a + b) = \alpha a + \alpha b$$
$$a \cdot \alpha \circ b = a\alpha - b = \alpha a - b = \alpha \cdot a \circ b\ .$$

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