SPECTRAL DECOMPOSITION OF A CLASS OF OPERATORS

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1. Introduction. Recently there has been a great interest in the study of various classes of operators on a complex Banach space with the purpose of developing a spectral theory generalizing that of self-adjoint operators on Hilbert space. Most well-known is the work of N. Dunford [2,3] on spectral operators. Many operators of interest are ruled out of this class by the requirement that there exist a countably additive resolution of the identity; for example, U. Fixman [5] has given examples of invertible isometries of $L_p(p \neq 2)$ and $C(X)$ which fail to be spectral. This present work is directed toward the development of a spectral theory where the resolution of the identity need only be finitely additive.

The general method is to use the classical operational calculus,

$$\varphi: f \to f(T) = \frac{1}{2\pi i}\int_\gamma f(z)R(z;\ T)dz\ ,$$

to induce a ring homomorphism of $\mathfrak{F}(T)$, the ring of all functions analytic on neighborhoods of $\sigma(T)$. This calculus may be used to induce a norm on $\mathfrak{F}(T)$ by defining $|f|$ to be $|f(T)|$. We may complete $\mathfrak{F}(T)$ in this norm to obtain a Banach algebra A which includes $\mathfrak{F}(T)$ as a dense subspace and on which there is a unique continuous extension of the classical homomorphism. Alternatively, under certain hypotheses, we may equip $\mathfrak{F}(T)$ with a norm so that the completion in this norm yields a regular function algebra.

What we will need, and therefore hypothesize, is a norm that accomplishes both conditions, namely, the operational calculus, φ, is to be bounded on $\mathfrak{F}(T)$ and the completion of $\mathfrak{F}(T)$ is to be a regular function algebra. We will need to impose further conditions at the "wrong end" of the theory; it will not be obvious from knowledge of an operator whether these conditions are satisfied. It will be clear, however, that the operationally defined class of operators contains some more easily indentified classes. We also impose a "front end" restriction on the spectrum of the operator, the R-set property.

In § 2, we obtain the consequences of the existence of an opera-

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