SILOV TYPE C ALGEBRAS OVER A CONNECTED LOCALLY COMPACT ABELIAN GROUP II

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In 1951 Silov [6] published a structure theory for a class of translation invariant function algebras on compact abelian groups. In 1959 the author extended portions of this structure theory to similar algebras defined on connected locally compact abelian groups [8]. One of the conditions which both Silov and the author employed was that all of the maximal regular ideals of the algebra be determined by the elements of the underlying groups in the usual way. In 1958 de Leeuw [2] published results characterizing the maximal ideals of an algebra of functions on a compact abelian group which satisfies all of Silov's conditions except this one. The results to be reported here constitute, in effect, an additional chapter to [8] motivated by an attempt to generalize de Leeuw's results. We will adopt de Leeuw's terminology, calling an algebra of the type studied in [6] and [8] a Silov-homogeneous algebra and an algebra which satisfies the weakened conditions of de Leeuw and the present paper a homogeneous algebra.

1. It is appropriate to begin with a brief discussion of an example of a Silov-homogeneous algebra which is a generalization of the group algebra of a locally compact abelian group. Domar, Beurling, Wermer and others have studied algebras of this type and we shall refer to results of Domar [3] in this connection. It is also an example which can be generalized in a natural way to include algebras of the type which we wish to discuss here and for which our results take a particularly simple form.

Let $G = \{s, t, \dots\}$ be a locally compact abelian group and let $\hat{G} = \{\chi, \dots\}$ be the group of characters of G. Suppose that p is a real measurable function on \hat{G} which is bounded on compact sets and satisfies the conditions

$$(1.1) p(\chi) \ge 1$$

$$(1.2) p(\chi_1\chi_2) \leq p(\chi_1)p(\chi_2)$$

(1.3)
$$\sum \frac{1}{n^2} \log p(\chi^n) < \infty$$

for all χ , χ_1 , $\chi_2 \in \widehat{G}$.

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