PACIFIC JOURNAL OF MATHEMATICS
Vol. 15, No. 1, 1965

THE COLLINEATION GROUPS OF DIVISION RING PLANES II: JORDAN DIVISION RINGS

R. OEHMKE AND R. SANDLER

In this paper the authors continue their study of the collineation groups of division ring planes (The collineation groups of division ring planes I. Jordan division algebras, J. Reine and Angew. Math. vol. 216, 1964). Some of the results obtained for finite dimensional Jordan division algebras are extended to a special class of infinite dimensional algebras.

As is well-known the study of the collineation group of a projective plane π coordinatized by an algebra $\mathscr{R}$ can be reduced to the study of the autotopism group of $\mathscr{R}$ or the group of autotopic collineations of π, $\mathscr{H}(\pi)$. The pair (a, b), $a, b \in \mathscr{R}$, is defined to be admissible if and only if there exists an element α in $\mathscr{H}(\pi)$ with $(1, 1)\alpha = (a, b)$. Modulo the automorphism group of $\mathscr{R}$, the determination of $\mathscr{H}(\pi)$ is equivalent to the determination of all admissible pairs (a, b) and coset representatives $\varphi_{a,b} \in \mathscr{H}(\pi)$ such that $(1, 1)\varphi_{a,b} = (a, b)$. With either the assumption $\mathscr{R}$ algebraic over its center, or the assumptions characteristic of $\mathscr{R}$ not equal to 0 and the centers of $\mathscr{R}$ and $\mathscr{R}'$ (the algebra of all elements of $\mathscr{R}$ algebraic over the center of $\mathscr{R}$) equal, the admissible pairs (a, b) are determined. Use is made of Kleinfeld's result on the middle nucleus of Jordan rings (Middle nucleus = center in a simple Jordan ring, to appear.) We also prove and use the result that the algebra $\mathscr{E}$ consisting of all right multiplications R_f is commutative, where f is in the subalgebra generated by a and a^{-1} over the base field.

Let $\mathfrak{R}$ be any nonalternative division ring (i.e., $(\mathfrak{R} - \{0\}, \cdot)$ is a loop), and let $\pi(\mathfrak{R})$ be the projective plane coordinatized by $\mathfrak{R}$. Then, as is well known, the study of the collineation group of π, $G(\pi)$, can be reduced to the study of the autotopism group of $\mathfrak{R}$, or the group of autotopic collineations of π, $H(\pi)$. If α is a collineation of π, then $\alpha \in H(\pi)$ if and only if $(\infty)\alpha = (\infty)$, $(0)\alpha = (0)$, $(0, 0)\alpha = (0, 0)$. Now, in [3], the pair (a, b) was defined to be admissible if and only if there exists an element $\alpha \in H(\pi)$ with $(1, 1)\alpha = (a, \mathrm{b})$, and it was shown that, modulo the automorphism group of $\mathfrak{R}$, $H_1(\mathfrak{R})$, the determination of $H(\pi)$ is equivalent to the determination of all admissible pairs (a, b) and coset representatives $\varphi_{a,b} \in H(\pi)$:

$$(1) \qquad (1, 1)\varphi_{a,b} = (a, b) .$$

Received January 23, 1964.