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REMARKS ON SIMPLE EXTENDED LIE ALGEBRAS

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We continue the discussion of finite dimensional simple extended Lie algebras over an algebraically closed field F of characteristic zero with nondegenerate form $(x, y) =$ trace $R_x R_y$ where R_x (or $R(x)$) denotes the mapping $A \rightarrow A: a \rightarrow ax$; for brevity we call such an algebra a *simple el-algebra*. The main result of this paper is that those simple el-algebras which are not Lie or Malcev algebras probably cannot be analyzed by the usual desirable Lie-type methods.

First if we assume the simple el-algebra [3] A has a diagonalizable Cartan subalgebra [3] such that for any weight space $A(N, \alpha)$ of N in A we have $A(N, \alpha)^2 = 0$ or $A(N, \alpha)^2 \subset A(N, \beta)$ for some weight β (which is a function of α), then A is a Lie or Malcev algebra. Thus if one attempts to remedy the situation that $A(N, \alpha)^2$ is difficult to locate by the rather desirable above assumptions and tries to construct a multiplication table for a new simple el-algebra, then actually nothing new is obtained. Next we show that if the derivation algebra $D(A)$ is used to analyze a simple el-algebra, using [1, page 54] or possibly Lie module theory, then again a difficult situation is encountered: If A is simple el-algebra, then A is not a simple Lie or Malcev algebra if and only if there exists a nonzero element $a \in A$ such that for every derivation $D \in D(A)$ we have $aD = 0$. The element $a \in A$ reflects the structure of A and so it appears that the structure of A is not accurately reflected in its derivation albgebra.

The proofs of the above results use the following lemma.

LEMMA 1.1. *If A is a simple el-algebra, then A is a Lie or 7-dimensional Malcev algebra if and only if $u(x) =$ trace R_x is the zero linear functional.*

Proof. A linearization of the defining identities of an extended Lie algebra

$$xy = -yx \qquad \text{and} \qquad J(xy, x, y) = 0$$

where $J(x, y, z) = xy \cdot z + yz \cdot x + zx \cdot y$ yields

$$J(wx, y, z) + J(yz, w, x) = J(wy, z, x) + J(zx, w, y) \tag{1.2}$$

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