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HOPF ALGEBRAS OVER DEDEKIND DOMAINS AND TORSION IN H-SPACES

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The main purpose of this note is to show that if the loop space ΩX of a finite dimensional H-space is free of torsion, then X itself can have p-torsion of at most order p.

§1 is devoted to proving a generalization to Dedekind domains of the decomposition theorems Hopf-Leray, and Borel, and §2 is devoted to recalling the structure of quasimonogenic Hopf algebras over the integers as described by Halpern. §3 gives the proof of the main theorem which relies somewhat on the statement and proof of Theorem 4.1 of [4].

Theorem 1.5 was included in the author's dissertation (Princeton University, 1961) done under the direction of Professor John Moore.

1. Hopf algebras over Dedekind domains.

Unless further specified K will denote an arbitrary integral domain. By *standard field associated* with K we shall mean any residue class field of K. A K-algebra will be called *monogenic* if it is generated by a single element.

In this section we prove a generalization (Theorem 1.5) for torsion-free algebras over a Dedekind domain with perfect quotient field of the following well known theorem:

THEOREM 1.1. (*Hopf-Leray-Borel*). *If B is a connected, commutative, and associative Hopf algebra of finite type over a perfect field K, then B is isomorphic as a K-algebra to a tensor product of monogenic Hopf algebras over K.*

Proof of the separate cases char $K = 0$ (Hopf-Leray) and char $K \neq 0$ (Borel) may be found in Milnor and Moore [6].

DEFINITIONS. A *closed submodule* of a K-module B is a submodule such that for all $x \in B$ and all $k \in K$, $kx \in A$ implies $x \in A$ or $k = 0$. If A is any submodule of B, then $\bar{A}$, the *closure of A in B*, is given by

$$\bar{A} = \{x \in B \mid kx \in A \text{ for some } k \in K,\ k \neq 0\}$$

REMARKS. A is closed in B is equivalent to $\bar{A} = A$ and to B/A is torsion-free. Note that the intersection of closed submodules is closed

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