PACIFIC JOURNAL OF MATHEMATICS
Vol. 15, No. 2, 1965

ON A CLASS OF CAUCHY EXPONENTIAL SERIES

J. A. ANDERSON AND G. H. FULLERTON

This paper was received before the synoptic introduction became a requirement.

1. **Introduction.** Let $Q(z)$ be a meromorphic function with poles $z_1, z_2, z_3, \cdots$, the notation being so chosen that $|z_1| \leqq |z_2| \leqq |z_3| \leqq \cdots$. If $f \in L(0, 1)$, define

$$c_\nu e^{z_\nu x} = \operatorname{res}_{z_\nu} Q(z) \int_0^1 f(t) e^{z(x-t)} dt .$$

Then, the series $\Sigma c_\nu e^{z_\nu x}$ is called the Cauchy Exponential Series (CES) of f with respect to $Q(z)$. If z_ν is of multiplicity m, then c_ν is a polynomial in x of degree at most $m-1$; if the poles are all simple, with residue λ_ν at z_ν, we may write

$$c_\nu = \lambda_\nu \int_0^1 f(t) e^{-z_\nu t} dt \tag{1}$$

and $\{c_\nu\}$, independent of x, are called the CE constants.

Let $C_p \colon |z| = r_p$ be an expanding sequence of contours, none of which passes through a pole of $Q(z)$. Suppose C_p contains n_p poles of $Q(z)$. Then,

$$\begin{aligned} \sum_{\nu=1}^{n_p} c_\nu e^{z_\nu x} &= \frac{1}{2\pi i} \int_{C_p} Q(z) dz \int_0^1 f(t) e^{z(x-t)} dt , \\ &= I_p , \quad \text{say} . \end{aligned}$$

Denote by C_p^+, C_p^- the parts of C_p lying in the right, left half-planes respectively. If $Q(z)$ is approximately unity on C_p^+, and is small on C_p^-, in the sense that

$$\int_{C_p^+} (Q(z) - 1) dz \int_0^1 f(t) e^{z(x-t)} dt = o(1) \tag{2}$$

$$\int_{C_p^-} Q(z) dz \int_0^1 f(t) e^{z(x-t)} dt = o(1) \tag{3}$$

as $p \to \infty$, uniformly for $x \in [0, 1]$, then

$$\begin{aligned} I_p &= \frac{1}{2\pi i} \int_{C_p^+} dz \int_0^1 f(t) e^{z(x-t)} dt + o(1) \\ &= \frac{1}{\pi} \int_0^1 \frac{f(t) \sin r_p(x-t)}{x-t} dt + o(1) \end{aligned}$$

Received March 4, 1963.