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LIE AND JORDAN STRUCTURES IN BANACH ALGEBRAS

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We first consider the theory of Jordan homomorphisms and Jordan ideals in Banach algebras. If B is a B*-algebra or a semi-simple annihilator algebra, any closed Jordan ideal in B is a two-sided ideal. Any Jordan homomorphism of a Banach algebra onto B is automatically continuous. That Jordan homomorphisms are continuous and Jordan ideals are ideals is shown to hold in a number of other situations. We also study the Lie ideals in a semi-simple Banach algebra A. If the center of A is zero and proper closed Lie ideals do not contain their Lie annihilators, then A is direct topological sum of its minimal closed ideals. An H^*-algebra with zero center is an example of such an algebra.

The utility of the study of Jordan isomorphisms in Banach algebra was noted by Kadison [8] in the study of isometrics of B^*-algebras. The Jordan and Lie structures of simple associative rings has been investigated by Herstein in a series of paper (see [3], [4], [5]). Essential use is made of these results in the present work.

2. **Pure algebra.** Let R be an associative ring. As is well-known [3] we can make R into a Jordan (Lie) ring by introducing the Jordan (Lie) multiplication $x\cdot y = xy + yx([x, y] = xy - yx)$. For a subset S of R we consider the sets

$$S^J = \{x \in R \mid x\cdot u = 0 \text{ for all } u \in S\}\ ,$$
$$S^L = \{x \in R \mid [x, u] = 0 \text{ for all } u \in S\}\ ,$$
$$\mathfrak{R}(S) = \{x \in R \mid ux = 0 \text{ for all } u \in S\} \text{ and}$$
$$\mathfrak{L}(S) = \{x \in R \mid xu = 0 \text{ for all } u \in S\}\ .$$

By an *ideal* in R we mean, unless otherwise specified, a two-sided ideal.

2.1. LEMMA. *Let U be a Lie ideal in R. Then U^J and U^L are Lie ideals.*

Let $x \in U^J$, $u \in U$ and $b \in R$. Since $xu = -ux$, an easy computation shows that $[x, b]\cdot u = [b, u]\cdot x = 0$. Let $y \in U^L$. Since $yu = uy$, we obtain by straightforward calculation that $[[y, b], u] = [y, [b, u]] = 0$.

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