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DEVELOPMENT OF THE MAPPING FUNCTION AT A CORNER

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Let D be a domain in the plane which is partially bounded by two curves Γ_1 and Γ_2 which meet at the origin and form there an interior angle $\pi\tau > 0$. Let N be an integer $\geqq 2$ and let α be a real number such that $0 < \alpha < 1$. Suppose that for $i = 1, 2, \Gamma_i$ admits a parametrization $x = x_i(t), y = y_i(t), 0 \leqq t \leqq 1$, where x_i and y_i have Nth derivatives which are uniformly α-Hölder continuous, and $|x_i'(t)| + |y_i'(t)| > 0$. Let $F(z)$ map the upper half plane conformally onto D in such a way that $F(0) = 0$. Then if τ is irrational $F(z)$ has an asymptotic expansion in powers of z and z^τ, with error term $o(z^{N\tau-\varepsilon})$. If $\tau = p/q$, a reduced fraction, then $F(z)$ has an asymptotic expansion in powers of z, z^τ, and $z^p \log z$, with error term $o(z^{N\tau-\varepsilon})$. In both cases ε is an arbitrarily small positive number. Furthermore expansions for derivatives of $F(z)$ of order $\leqq N$ may be obtained by differentiating formally.

The behavior of such conformal maps at corners was first investigated by Lichtenstein [9]. Let $F^{-1}(z)$ be the function inverse to $F(z)$ which maps D onto the upper half plane. Lichtenstein showed that if Γ_1 and Γ_2 are analytic then

$$\frac{d}{dz}F^{-1}(z) = z^{1/\tau-1}\varphi(z) \tag{1.1}$$

where $\varphi(z)$ is continuous in $\bar{D}$ and $\varphi(0) \neq 0$. This result was later generalized in two ways. One was to weaken the requirements on Γ_1 and Γ_2. It follows from the work of Kellogg [4] and Warschawski [10] that with very modest conditions imposed on Γ_1 and Γ_2 one has

$$F^{-1}(z) = z^{1/\tau}\varphi(z)$$

where again $\varphi(z)$ is continuous in $\bar{D}$ and $\varphi(0) \neq 0$. In particular this follows if one assumes that Γ_1 and Γ_2 have continuously turning tangents in a neighborhood of the origin (though weaker conditions will suffice).

The other generalization of Lichtenstein's theorem was an improvement of the result (1.1), maintaining the analyticity requirement. For the case $\tau = 1$ Lewy [8] showed that $F(z)$ has an asymptotic expansion

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