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GENERALIZED CHARACTER SEMIGROUPS: THE SCHWARZ DECOMPOSITION

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The author's résumé: A structure theorem due to Š. Schwarz asserts that if S is a finite abelian or a compact abelian semigroup admitting relative inverses, than the character semigroup of S is decomposed into a disjoint union of character groups of certain maximal subgroups of S. In this note, among other things, we generalize this Schwarz Decomposition Theorem to a broader class of semigroups, the so-called pseudo-invertible semigroups. We also relax the range of the characters from the semigroup of complex numbers to a more general semigroup.

For notations and terms not defined here see A. D. Wallace [11]. *Throughout this paper, let S be always a compact commutative semigroup, unless otherwise stated.* By a *character* of S is meant a continuous homomorphism of S into the multiplicative semigroup C of the complex numbers endowed with the usual Euclidean topology. The collection of all characters of S, with the value-wise multiplication of functions, endowed with the compact-open topology, forms a semigroup which will be denoted by $(S, C)^\wedge$ or simply $S^\wedge$, and will be called the *character semigroup* of S. Hewitt and Zuckerman [4] use the term *semicharacter*, in the discrete case, for not identically zero characters. Here we use $(\hat{S}, C)$ or simply $\hat{S}$, as distinguished from $S^\wedge$, to denote the collection of semicharacters of S. We note that $\hat{S}$, in general, need not be a semigroup. We first draw attention to the fact that if χ is a character of S, then $|\chi(x)| \leqq 1$ for every x in S. For, otherwise $\chi(S)$ would not be compact. Thus, in the study of characters, only the unit disc $\{z: |z| \leqq 1\}$ of the complex numbers is used. Let us write D for this unit disc. The set D itself forms an important semigroup which is compact, connected, commutative, cancellable,[1] has zero 0 and unit 1; moreover the circumference $\{z: |z| = 1\}$ of D is the maximal subgroup $H(1)$ and $D \backslash H(1)$ is an ideal. However, only some of these are needed as we shall see below.

Throughout the rest of this paper, let T be an arbitrary, but fixed, compact commutative cancellable semigroup with zero z and unit u[2] such that $T \backslash H(u)$ is a subsemigroup of T. By a *generalized character*

[1] A semigroup S is cancellable if and only if for any nonzero elements a, b, c in S such that $ab = ac$ or $ba = ca$, then $b = c$.

[2] It is to be understood that $z \neq u$.

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