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A THEOREM ON PARTITIONS OF MASS-DISTRIBUTION

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A 'bisector' of a continuous mass-distribution M in a bounded region on the plane is defined as a straight line such that the two half-planes determined by this line contain half the mass of M each. It is known that there exists at least one point (in the plane) through which pass three bisectors of M.

THEOREM. **Let, for a continuous mass distribution M, the point P through which three bisectors pass be unique. Then all bisectors of M pass through p.**

The following corollary also is established: For a convex figure K (i.e., compact convex set with nonempty interior) to be centrally symmetric, it is necessary and sufficient that the point through which three bisectors of area pass be unique.

In what follows, M stands for any continuous mass-distribution in a compact domain in the plane. A line l is called a *bisector* of M if the two half-planes determined by l contain equal masses of M.

The following results are well-known regarding bisectors of M. (see, for example, [4], Problem 3–1, 3–2, and [1]).

(1) Let l be any line in the plane. There is a bisector of M parallel to l.

(2) There exists a point P in the plane and two perpendicular lines through P such that the portions of M contained in each of the four 'wedges' determined by these lines have the same mass, namely, a quarter of that of M.

(3) There exists a point in the plane through which three distinct bisectors of M pass.

Further, let l_0 be a bisector of M and 0 a fixed point on l_0. Let $l(\alpha)$ be a bisector of M, inclined to l_0 at an angle α and intersecting l_0 in P_α. It is easy to verify that we can choose the bisector $l(\alpha)$ such that the distance $0P_\alpha$ is a continuous function of α. We shall make use of this observation in the following.

In this paper we shall investigate the nature of the points through which three distinct bisectors of M pass. Specifically, let P be a point

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