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A THEOREM OF LITTLEWOOD AND LACUNARY SERIES FOR COMPACT GROUPS

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Let G be a compact group and $f \in L^2(G)$. We prove that given $p < \infty$ there exists a unitary transformation U of $L^2(G)$ into $L^2(G)$, which commutes with left translations and such that $Uf \in L^p$. The proof is based on techniques developed by S. Helgason for a similar question. The result stated above, which is an extension of a theorem of Littlewood for the unit circle is then applied to the study of lacunary Fourier series.

The following two results concerning Fourier series of functions defined on the unit circle were proved by Littlewood [5]:

I. *Suppose that for any choice of complex numbers α_n, with $|\alpha_n| = 1$, $\sum \alpha_n a_n e^{inx}$ is the Fourier series of an integrable function (or a Fourier-Stieltjes series) then $\sum |a_n|^2 < \infty$.*

II. *Let $\sum |a_n|^2 < \infty$. Then given $p < \infty$ there exist complex numbers α_n, with $|\alpha_n| = 1$, such that $\sum \alpha_n a_n e^{inx}$ is the Fourier series of a function in L^p.*

Helgason [3] has generalized I to Fourier series on compact groups. Let G be a compact group with normalized Haar measure dx. If $f \in L^1(G)$ then f is uniquely represented by a Fourier series

$$f(x) \sim \sum_{\gamma \in \Gamma} d_\gamma Tr(A_\gamma D_\gamma(x))$$

where Tr denotes the usual trace, Γ is the set of equivalence classes of irreducible unitary representations of G, D_γ is a representative of the class γ, d_γ is the degree of γ, and A_γ is the linear transformation given by

$$A_\gamma = \int_G f(x) D_\gamma(x^{-1}) dx .$$

Helgason has proved

I′. *Suppose that, for any choice of unitary transformations U_γ on the Hilbert space of dimension d_γ, $\sum_{\gamma \in \Gamma} d_\gamma Tr(U_\gamma A_\gamma D_\gamma(x))$ is the Fourier series of an integrable function (or a Fourier-Stieltjes series) then*

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