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ALGEBRAS AND FIBER BUNDLES

J. M. G. FELL

Let A be an associative algebra and $\hat{A}_n$ the family of all equivalence classes of irreducible representations of A of dimension exactly n. Topologizing $\hat{A}_n$ as in a paper about to appear in the Transactions of the American Mathematical Society, we show that for each n, A gives rise to a fiber bundle having $\hat{A}_n$ as its base space and the $n \times n$ total matrix algebra as its fiber.

Throughout this note A will be an arbitrary fixed associative algebra over the complex field C. By a *representation* of A we understand a homomorphism T of A into the algebra of all linear endomorphisms of some complex linear space $H(T)$, the *space of* T. We write $\dim(T)$ for the dimension of $H(T)$. Irreducibility and equivalence of representations are understood in the purely algebraic sense. If T is a representation, $r \cdot T$ will be the direct sum of r copies of T. Let $\hat{A}^{(f)}$ the family of all equivalence classes of finite-dimensional irreducible representations of A; and put

$$\hat{A}^{(n)} = \{T \in \hat{A}^{(f)} \mid \dim(T) \leqq n\},\ \hat{A}_n = \{T \in \hat{A}^{(f)} \mid \dim(T) = n\} .$$

We shall usually not distinguish between representations and the equivalence classes to which they belong.

Let T be a finite-dimensional representation of A. If for each a in A $\tau(a)$ is the matrix of T_a with respect to some fixed ordered basis of $H(T)$, then $\tau : a \to \tau(a)$ is a *matrix representation of* A *equivalent to* T.

By $A^{\#}$ we mean the space of all complex linear functionals on A, and by $\operatorname{Ker}(\varphi)$ the kernel of φ. If $T \in \hat{A}^{(f)}$, we put

$$\Phi(T) = \{\varphi \in A^{\#} \mid \operatorname{Ker}(T) \subset \operatorname{Ker}(\varphi)\} .$$

An element φ of $A^{\#}$ is *associated* with T if $\varphi \in \Phi(T)$. One element of $\Phi(T)$ is of course the character χ^T of $T(\chi^T(a) = \operatorname{Trace}(T_a)$ for a in A). An element T of $\hat{A}^{(f)}$ is uniquely determined by the knowledge of one nonzero functional in $\Phi(T)$ ([2], Proposition 2).

As in [2] we equip $\hat{A}^{(f)}$ with the *functional topology* as follows: If $T \in \hat{A}^{(f)}$ and $\mathscr{S} \subset \hat{A}^{(f)}$, T belongs to the functional closure of $\mathscr{S}$ if $\Phi(T) \subset (\bigcup_{S \in \mathscr{S}} \Phi(S))^-$ where $^-$ denotes closure in the topology of pointwise convergence on A.

Our main object in this note is to prove the following fact about

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