PACIFIC JOURNAL OF MATHEMATICS
Vol. 17, No. 1, 1966

INTEGRAL SOLUTIONS TO THE INCIDENCE EQUATION FOR FINITE PROJECTIVE PLANE CASES OF ORDERS $n \equiv 2 \pmod 4$

E. C. JOHNSEN

A finite projective plane of order $n \geqq 2$ can be considered as a $\langle v, k, \lambda \rangle$ design where $v = n^2 + n + 1$, $k = n + 1$, and $\lambda = 1$. As such, it can be characterized by its point-line 0, 1 incidence matrix A of order v satisfying the incidence equation

$$(*) \qquad AA^T = nI + J,$$

where J is the matrix of order v consisting entirely of 1's. Thus, if a plane of order n exists then (*) has an integral solution A. Ryser has shown that if A is a normal integral solution to (*) or if A is merely an integral solution to (*) where n is odd, then A can be made into an incidence matrix for a plane of order n by suitably multiplying its columns by -1. Such an integral solution to (*) we shall call a type I solution. When A is merely an integral solution to (*) where n is even, then A may be a type I solution but may also be not of this type. These latter integral solutions to (*) we shall call type II solutions. Ryser has constructed type II solutions for $n = 2$ and for all $n \equiv 0 \pmod 4$ for which there exists a Hadamard matrix of order n, and Hall and Ryser have constructed a type II solution for $n = 10$. In this paper we construct type II solutions for some infinite classes of values of $n \equiv 2 \pmod 4$. Basic to these constructions is a special class of $\langle v, k, \lambda \rangle$ designs called skew-Hadamard designs whose incidence matrices form a part of the substructure of our type II solutions. We exhibit examples for $n = 26$ and 50 and also derive examples for $n = 10$ and 18.

A $\langle v, k, \lambda \rangle$ *design* is an arrangement of v elements $x_1, x_2, \cdots, x_v$ into v sets $S_1, S_2, \cdots, S_v$ such that every set contains exactly k elements, every pair of sets has exactly λ elements in common, and to avoid certain degenerate situations, $0 \leqq \lambda < k \leqq v - 1$. A $\langle v, k, \lambda \rangle$ design can be characterized by its *incidence matrix* $A = [a_{ij}]$ by writing the elements $x_1, x_2, \cdots, x_v$ in a row and the sets $S_1, S_2, \cdots, S_v$ in a column and setting $a_{ij} = 1$ if $x_j \in S_i$ and $a_{ij} = 0$ if $x_j \notin S_i$. This matrix A, of order v, consists entirely of 0's and 1's and, by the conditions given above, is easily seen to satisfy the *incidence equation*:

Received, December 22, 1964, in revised form, May 18, 1965. The material in this paper is part of the author's doctoral dissertation written at The Ohio State University in 1961, the research for which was supported by the Office of Ordnance Research.