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ON STRATIFIABLE SPACES

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In the enclosed paper, it is shown that (a) the closed continuous image of a stratifiable space is stratifiable (b) the well-known extension theorem of Dugundji remains valid for stratifiable spaces (see Theorem 4.1, Pacific J. Math., 1 (1951), 353-367) (c) stratifiable spaces can be completely characterized in terms of continuous real-valued functions (d) the adjunction space of two stratifiable spaces is stratifiable (e) a topological space is stratifiable if and only if it is dominated by a collection of stratifiable subsets (f) a stratifiable space is metrizable if and only if it can be mapped to a metrizable space by a perfect map.

In [4], J. G. Ceder studied various classes of topological spaces, called M_i-spaces ($i = 1, 2, 3$), obtaining excellent results, but leaving questions of major importance without satisfactory solutions. Here we propose to solve, in full generality, two of the most important questions to which he gave partial solutions (see Theorems 3.2 and 7.6 in [4]), as well as obtain new results.[1] We will thus establish that Ceder's M_3-spaces are important enough to deserve a better name and we propose to call them, henceforth, STRATIFIABLE spaces. Since we will exclusively work with stratifiable spaces, we now exhibit their definition.

DEFINITION 1.1. A topological space X is a *stratifiable* space if X is T_1 and, to each open $U \subset X$, one can assign a sequence $\{U_n\}_{n=1}^{\infty}$ of open subsets of X such that

(a) $U_n^- \subset U$,
(b) $\bigcup_{n=1}^{\infty} U_n = U$,
(c) $U_n \subset V_n$ whenever $U \subset V$.

For convenience, we will say that $\{U_n\}$ (more precisely, $\{U_n\}_{n=1}^{\infty}$) is a *stratification of* U whenever the U_n satisfy (a) and (b) of Definition 1.1. Similarly, we will say that the correspondence $U \rightarrow \{U_n\}$ is a *stratification of* X whenever the U_n satisfy (a), (b) and (c) of Definition 1.1. Certainly, we may suppose that any stratification $U \rightarrow \{U_n\}$ of X is *increasing*, i.e. $U_n \subset U_{n+1}$ for each n (if $U \rightarrow \{U_n\}$ is a stratification of X, then so is $U \rightarrow \{U'_n\}$, where $U'_n = \bigcup_{i=1}^{n} U_i$), a fact that will actually be used in § 4. The same applies to stratifications of

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