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CAUCHY SEQUENCES IN MOORE SPACES

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In this paper the concept of a Cauchy sequence is extended to Moore spaces. This concept is then used to characterize those Moore spaces which are completable (i.e., topologically equivalent to a subspace of a complete Moore space). The definition for Cauchy sequence given here is shown to be a generalization of the usual definition for Cauchy sequence in a metric space. Also considered are certain questions concerning completability of a Moore space having a dense completable subspace.

It is well known that every metric space is a subspace of a complete metric space. In [6] Mary Ellen Rudin proved the existence of Moore spaces that are not subspaces of complete Moore spaces. In [1] O. H. Alzoobaee gave a sufficient condition that a Moore space be a subspace of a complete Moore space, but it was not stated and it appears to be unknown whether this condition is necessary. The results of the present paper were obtained independently of Alzoobaee's paper.

Let S be a topological space and let G be a monotonically decreasing sequence of open coverings of S. The statement that G is a *development* for S means that if D is an open set containing the point x, then there exists a positive integer n such that if $R \in G_n$ $x \in R$ then $\bar{R} \subset D$. A topological (T_1) space S having a development is called a *Moore space*. The statement that G is a *strong development* for S means that if D is an open set containing the point x, then there exists a positive integer n such that if $R \in G_n$ and $x \in \bar{R}$ then $\bar{R} \subset D$.

If G is a development for the Moore space S and if H is a monotonically decreasing sequence of open coverings of S such that H_n is a refinement of G_n for each n, then H is said to be a *refinement* of G. A refinement of a development is a development, and a refinement of a strong development is a strong development. Furthermore, every development has a refinement that is a strong development.

The statement that a development G for S is *complete* means that if T is a monotonically decreasing sequence of closed point sets such that T_n is contained in an element of G_n for each n, then there is a point x such that $x \in T_n$ for each n. A Moore space is said to be *complete* if it has a complete development. In [6, Th. 3, p. 322], M. E. Rudin proved that a topological (T_1) space satisfies R. L. Moore's Axiom 1 [3] if and only if it has a complete development as defined