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SOME CLASSES OF RING-LOGICS

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Let $(R, \times, +)$ be a commutative ring with identity, and let $K = \{\rho_1, \rho_2, \cdots\}$ be a transformation group in R. The K-logic of the ring $(R, \times, +)$ is the (operationally closed) system $(R, \times, \rho_1, \rho_2, \cdots)$ whose operations are the ring product "$\times$" together with the unary operations $\rho_1, \rho_2, \cdots$ of K. The ring $(R, \times, +)$ is essentially a ring-logic, mod K, if the "$+$" of the ring is equationally definable in terms of its K-logic $(R, \times, \rho_1, \rho_2, \cdots)$. Our present object, is to show that any finite direct product of (not necessarilly finite) direct powers of finite commutative local rings of distinct orders is a ring-logic modulo certain suitably chosen (but nevertheless still rather general) groups. This theorem subsumes and generalizes Foster's results for Boolean rings, p-rings, and p^k-rings, as well as the author's results for residue class rings and finite commutative rings with zero radical. Several new classes of ring-logics (modulo certain groups of quite general nature) are also explicitly exhibited. Throughout the entire paper, all rings under consideration are assumed to be commutative and with identity.

The one component case. In this section, we direct special attention to arbitrary direct powers of a finite local ring in regard to the concept of ring-logics. First, we recall the following [9; 228]

DEFINITION 1. A ring R is called a *local ring* if and only if R is Noetherian and the nonunits of R form an ideal.

REMARK. It can be easily shown that for a finite commutative ring R with identity $1 (1 \neq 0)$, the concepts "local ring", "primary ring", and "completely primary ring" are equivalent. This readily follows by recalling that a primary ring is a ring R with identity such that R/J is a simple ring satisfying the minimum condition for right ideals, while a completely primary ring is a ring R with identity such that R/J is a division ring. Here, J is the radical of R. Hence the results below still hold if we replace the local rings involved by primary rings or by completely primary rings.

A very useful result for our purposes is the following

LEMMA 2. *Let R be a finite ring with identity $1 (1 \neq 0)$. The ring R is a local ring if and only if every element of R is either a unit or is nilpotent.*