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A THEOREM ON ONE-TO-ONE MAPPINGS

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Let X be a locally connected generalized continuum with the property that the complement of each compact set has only one nonconditionally compact component. The author proves the following theorem. If f is a one-to-one mapping of X onto Euclidean 2-space, then f is a homeomorphism.

An example of K. Whyburn implies that if f is a one-to-one mapping of X onto Euclidean n-space ($n \geqq 3$), then X can have many nice properties any yet f need not be a homeomorphism. However the complement of a compact set in the domain space of his example may have more than one nonconditionally compact component.

It is interesting to note that a characterization of closed 2-cells in the plane is obtained in the course of proving the theorem.

Positive results in connection with the following problem would be useful in classifying mappings from a Euclidean space into itself. "What properties must a topological space X have before one can conclude that every one-to-one mapping f of X into a Euclidean space E^n of dimension n is a homeomorphism?" A very general theorem of this type was supposedly obtained in [2]. However, several counterexamples have been obtained which show the main theorems of [2] to be false. One of these is an example of K. Whyburn [6], which implies that if $n \geqq 3$, X may have many nice properties, yet f need not be a homeomorphism. We prove that if the Euclidean space has dimension two, the mapping f is onto, and X has appropriate properties, then f is indeed a homeomorphism. It is interesting to note that we assign a property to the space X which is not a property of the domain space of the example in [6].

2. Notation. A mapping is a continuous function. A generalized continuum is a connected, locally compact, separable metric space. The cyclic element theory used is that of reference [4]. A set A in a topological space is conditionally compact if its closure is a compact set. A dendrite is a compact locally connected generalized continuum containing no simple closed curve. A topological line is a homeomorphic image of the real line. A topological ray is a homeomorphic image of a ray in the real line.

3. Theorem and proof.

THEOREM. *Let X be a locally connected generalized continuum*