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POINTWISE BOUNDS FOR THE SECOND INITIAL-BOUNDARY VALUE PROBLEM OF PARABOLIC TYPE

F. J. BELLAR, JR.

By means of the divergence theorem and certain well-known inequalities, the author presents explicit a priori pointwise bounds for the solution of the linear and nonlinear second initial-boundary value problem of the parabolic type. The desired result is obtained by using the parabolic form of Green's second identity with an appropriately defined parametrix serving as the first function of the identity and the difference of the solution and an arbitrary function which approximates the given data as the second. By means of various well-known inequalities, the unknown integrals in the resulting expression are bounded in terms of volume and surface integrals of the square of known functions. In the linear case the form of the bound is such that it may be improved by employing the Rayleigh-Ritz technique.

Background. In [8], Payne and Weinberger presented a method for determining similar bounds for the solution of Dirichlet and mixed type boundary value problems for certain second order elliptic equations and rather general domains. The results were extended by Bramble and Payne [2] and by the author [1] to the Neumann Problem for second order uniformly elliptic operators. In sequels the author will consider the first initial-boundary value problem of the parabolic type, degenerate parabolic problems of various types and nonlinear elliptic problems.

The results presented herein are applicable to problems for which existence and uniqueness theorems have not as yet been established. Assuming the existence of a solution, its uniqueness follows immediately from the form of the pointwise bound.

Since bounds for various quadratic functionals are also obtained in terms of the given data, it is assumed that the results of this paper may profitably be employed in the derivation of such existence proofs. For information pertaining to parabolic problems for which solutions are known to exist, it is suggested that the reader review [7].

In § 2 the problem under consideration is defined in detail; certain auxiliary functions and other preliminary matters are considered in the next two sections. The pointwise bound is obtained in § 5, while the generalization to the nonlinear case is considered in the following section. The last section contains some additional remarks.