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THE EXPONENTIAL ANALOGUE OF A GENERALIZED WEIERSTRASS SERIES

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The generalized Weierstrass series

$$\sum_{n=1}^{\infty} a_n \frac{z^n}{1+z^{2n}}$$

has as its exponential analogue

$$\sum_{n=1}^{\infty} a_n \frac{e^{-\lambda_n z}}{1+e^{-2\lambda_n z}}$$

where $\{a_n\}$ is a sequence of complex-valued constants and $\{\lambda_n\}$ is any real-valued strictly monotone increasing unbounded sequence.

In this paper the λ_n will be chosen to be $\ln n$. Then the above series becomes

$$A(z) = \sum_{n=1}^{\infty} a_n \frac{n^{-z}}{1+n^{-2z}}, \tag{1}$$

hereafter called simply the A-series. In its region of absolute convergence an A-series can be expressed as a Dirichlet series; conversely, a Dirichlet series can be represented by an A-series. Under restrictions on the sequence $\{a_n\}$, the imaginary axis becomes a natural boundary of the function represented by the A-series.

Since $A(z) = A(-z)$, only values of $z = x + iy$, $x > 0$, will be considered. Similar results hold in (1) for corresponding values of $-z$. Hereafter, unless otherwise indicated, all summations will be understood to range from $n = 1$ to ∞.

2. Convergence of the A-series.

The following theorems on convergence are stated without proof.

THEOREM 1. (A) *If $\sum a_n$ diverges, the A-series converges and diverges for all points $z = x + iy$, $x > 0$, with the associated Dirichlet series $\sum a_n n^{-z}$.*

(B) *If $\sum a_n$ converges, the A-series converges for all points $z = x + iy$, $x > 0$.*

Theorem 1 remains true if ordinary convergence and divergence are replaced by absolute convergence and divergence throughout the statement of the theorem.