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POWER-ASSOCIATIVE ALGEBRAS IN WHICH EVERY SUBALGEBRA IS AN IDEAL

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By an H-algebra we mean a nonassociative algebra (not necessarily finite-dimensional) over a field in which every subalgebra is an ideal of the algebra.

In this paper we prove

MAIN THEOREM. **Let A be a power-associative algebra over a field F of characteristic not 2. A is an H-algebra if and only if A is one of the following;**

(1) a one-dimensional idempotent algebra;

(2) a zero algebra;

(3) an algebra with basis u_0, u_i, $i \in I$ (an index set of arbitrary cardinality) satisfying $u_i u_j = \alpha_{ij} u_0, \alpha_{ij} \in F$, $i, j \in I$, all other products zero. Moreover if J is a finite subset of I, then $\sum_{i,j \in J} \alpha_{ij} x_i x_j$ is nondegenerate in that $\sum_{i,j \in J} \alpha_{ij} \alpha_i \alpha_j = 0$, $\alpha_i, \alpha_j \in F$, $i \in J$ implies $\alpha_i = 0$, $i \in J$;

(4) direct sums of algebras of types (1), (2), (3) with at most one from each.

This is an extension of a result of Liu Shao-Xue who established it for alternative and Jordan H-algebras of characteristic not 2 [1; Theorem 1].

An immediate corollary is that a power-associative H-algebra over a field of characteristic not 2 is associative [1; Cor. 1].

Some results on H-rings are also determined in this paper. By an H-ring we mean a nonassociative ring in which every subring is an ideal.

1. Preliminaries.

The *associator* (x, y, z) is defined by $(x, y, z) = (xy)z - x(yz)$. We will use the *Teichmüller identity* which holds in an arbitrary ring,

$$(wx, y, z) - (w, xy, z) + (w, x, yz) = w(x, y, z) + (w, x, y)z \,. \tag{1.1}$$

In a power-associative ring we have the identities $(x, x, x) = 0$ and $(x^2, x, x) = 0$ which when linearized yield, respectively,

$$\sum_{\sigma \in S_3} (w_{\sigma(1)}, w_{\sigma(2)}, w_{\sigma(3)}) = 0 \tag{1.2}$$

and

$$\sum_{\sigma \in S_4} (w_{\sigma(1)} w_{\sigma(2)}, w_{\sigma(3)}, w_{\sigma(4)}) = 0 \tag{1.3}$$

providing $2x = 0$ implies $x = 0$ in the ring.