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ON THE CONVERGENCE OF RESOLVENTS OF OPERATORS

MINORU HASEGAWA

Let a family of linear operators $\{A_n\}(n = 1, 2, \cdots)$ in a Banach space X have the resolvents $\{R(\lambda; A_n)\}$ which is equicontinuous in n. Suppose that $\{A_n\}$ is a Cauchy sequence on a dense set. Then the question of convergence arises; when will $\{R(\lambda; A_n)x\}$ be a Cauchy sequence for all $x \in X$?

This problem is treated in some special cases and an application to the following theorem is presented.

Let A be the generator of a positive contraction semigroup $\sum$ and let B be a linear operator with domain $\mathscr{D}(B) \supset \mathscr{D}(A)$ in a weakly complete Banach lattice X.

Then $A + B$ or its closed extension generates a positive contraction semi-group $\sum'$ which dominates $\sum$ if and only if $A + B$ is dissipative and B is positive.

In this section we consider the above convergence problem in a Banach space X (cf. [9], [1], [11]).

Let a family of linear operators $\{A_n\}(n = 1, 2, \cdots)$ satisfy the following conditions:

(1) for some fixed number λ, the resolvent $R(\lambda; A_n) = (\lambda - A_n)^{-1}$ of A_n exists which acts on X to the domain $\mathscr{D}(A_n)$ of A_n and satisfies the norm condition $\| R(\lambda; A_n) \| \leqq K_\lambda$, where K_λ is a positive number independent of n,

(2) there is a dense subspace $\mathscr{M}$ on which $A = \lim A_n$ exists.

PROPOSITION 1. The limit operator $R_0(\lambda; A) = \lim R(\lambda; A_n)$ exists on $\overline{\mathscr{N}}$ and satisfies the norm condition $\| R_0(\lambda; A) \|_{\overline{\mathscr{N}}} \leqq K_\lambda$ where $\mathscr{N} = (\lambda - A)\mathscr{M}$ and $\overline{\mathscr{N}}$ is its closure.

Proof. For any $x \in \mathscr{M}$ we have

$$\| (\lambda - A_n)x \| \geqq K_\lambda^{-1} \| x \|$$

and thus obtain

$$\| (\lambda - A)x \| \geqq K_\lambda^{-1} \| x \| - \| A_n x - Ax \| .$$

Letting $n \rightarrow \infty$, we have

$$\| (\lambda - A)x \| \geqq K_\lambda^{-1} \| x \| .$$

It also follows that we can extend $(\lambda - A)^{-1}$ to the bounded linear operator $R_0(\lambda; A)$ on $\overline{\mathscr{N}}$ which satisfies