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EXISTENCE OF A HOMOTOPY OPERATOR FOR SPENCER'S SEQUENCE IN THE ANALYTIC CASE

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According to E. Cartan's prolongation theorem, an analytic system of linear partial differential equations becomes an involutive system, after prolongation in a finite number of steps, and an involutive system has local solutions, by the Cartan-Kähler theorem.

Recently, a homological procedure has been developed, in terms of which the notion of involution is equivalent to the vanishing of a certain type of cohomology (so-called "δ-cohomology"). Moreover, the local solvability of a linear system of partial differential equations has been shown by Quillen to be equivalent to the exactness, at degree one, of a certain resolution introduced originally by Spencer, which is canonically associated with the given system. The terms of the resolution are sheaves of germs of jet forms, i.e., differential forms with values in jet spaces.

The exactness of this resolution, providing a replacement for the Cartan-Kähler theorem in the linear case, in the analytic case is known. We shall have given another proof, based on the construction of a homotopy operator which is a natural generalization, to jet forms, of the well-known homotopy operator used in proving the Poincaré lemma for the exterior derivative d.

Some estimates will be necessary in order to study the existence of this operator, and we use here extensively the estimates obtained by Sweeney in [5] which are related to the bounds obtained earlier by L. Ehrenpreis, V. W. Guillemin and S. Sternberg in the paper [1].

1. Notation. Let M be a C^∞ manifold of real dimension n. Since everything we shall do is local, we shall always work in a neighborhood, U, of a reference point 0 of M. A coordinate $(x^1, x^2, \cdots, x^n)$, vanishing at 0, will be chosen in U.

If E is a C^∞ bundle over M, $\underline{E}$ denotes the sheaf of germs of C^∞ sections of E. But, for the sake of simplicity, and because E can be supposed trivial over U, we shall in general use the same notation E for both.

Also for the sake of simplicity, we shall use the following, now classical, condensed notations:

$p = (p_1, p_2, \cdots, p_n)$ is a multi-index of nonnegative integers p_i.