PACIFIC JOURNAL OF MATHEMATICS
Vol. 22, No. 2, 1967

FUNCTIONAL REPRESENTATION OF TOPOLOGICAL ALGEBRAS

PETER D. MORRIS AND DANIEL E. WULBERT

A topological algebra E is an algebra over the real or complex numbers together with a topology such that E is a topological vector space and such that multiplication in E is jointly continuous. For a topological space X, $C(X)$ denotes the algebra of all continuous, complex-valued functions on X with the usual pointwise operations. Unless otherwise stated, $C(X)$ is assumed to have the compact-open topology. Our principal concern is with representing (both topologically and algebraically) a commutative (complex) topological algebra, with identity, E as a subalgebra of some $C(X)$, X a completely regular Hausdorff space. We obtain several characterizations of topological algebras which can be so represented. The most interesting of these is that the topology on E be generated by a family of semi-norms each of which behaves, with respect to the multiplication in the algebra, like the norm in a (Banach) function algebra.

Let M be the set of nonzero, continuous, multiplicative, linear functionals on a topological algebra E, provided with the weak topology induced by E. We are especially interested in representing E as a subalgebra of $C(M)$. Our results along this line are found in § 4. If E is also provided with an involution, we wish to represent E in such a way that involution goes over into complex conjugation. This problem is studied in § 5.

The principal known results along the lines of our investigation are due to Arens and Michael. Arens ([3], Th. 11.4) characterized the topological algebras which are topologically isomorphic to $C(X)$, for X a paracompact space. Michael ([9], Th. 8.4, p. 33) obtained sufficient conditions for a topological algebra to be topologically isomorphic to $(C(M), T_0)$, where T_0 is a topology weaker than the compact-open topology. Arens ([3], Th. 11.6) obtained a similar result. We obtain Michael's result as Corollary 5.3.

In § 2 we prove that a Hausdorff space X is completely regular if and only if the closed ideals in $C(X)$ are in one-to-one correspondence (in the usual way) with the closed subsets of X. The necessity is well known in case X is compact but our theorem seems to be new. In § 3 we characterize those spaces $C(X)$ which have the Mackey topology. This section is unrelated to the rest of the paper but is of some interest in itself.

In § 6 we apply some of our previous results to the general study