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ABELIAN p-GROUPS DETERMINED BY THEIR ULM SEQUENCES

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Ulm's theorem asserts that, within the class of all reduced countable abelian p-groups, a group is determined, up to isomorphism, by its Ulm sequence. Although this theorem fails in general for uncountable groups, there are classes of uncountable abelian p-groups whose members are determined within the class by their Ulm sequences. Kolettis has shown that the class of direct sums of countable p-groups has this property. Here it is shown that the class of those abelian p-groups for which the Ulm type is finite and all the Ulm factors except the last are direct sums of cyclic groups, is another such class.

Let G be a reduced abelian p-group. Define the subgroup G^α for each ordinal α as follows. Set $G^0 = G$. Proceeding inductively, if $\alpha = \beta + 1$, define G^α to be the subgroup of those elements in G^β which have infinite height in G^β; if α is a limit ordinal, define $G^\alpha = \bigcap_{\beta<\alpha} G^\beta$. Since G is reduced, there is a first ordinal τ such that $G^\tau = 0$; this ordinal τ is called the *Ulm type* of G. The *Ulm factors* of G are defined to be the factor groups $G_\alpha = G^\alpha/G^{\alpha+1}$ $(\alpha > \tau)$. And the sequence of Ulm factors G_α $(\alpha < \tau)$ is called the *Ulm sequence* of G. Two groups G and H have *isomorphic Ulm sequences* if $G_\alpha \cong H_\alpha$ for every α.

Our theorem is now the following:[1]

If G is a reduced abelian p-group having finite Ulm type n and such that its first $n-1$ Ulm factors $G_0, \cdots, G_{n-2}$ are direct sums of cyclic groups, and if H is any other abelian p-group whose Ulm sequence is isomorphic to that of G, then $H \cong G$.

It should be noted that in this theorem no assumption is made on the last Ulm factor G_{n-1} of G.

Neither the assumption of finite Ulm type nor the assumption that all but the last Ulm factor are direct sums of cyclic groups can be dropped from the hypotheses of the theorem. For example, if G is any countable p-group whose Ulm type is infinite, then there is an uncountable p-group whose Ulm sequence is isomorphic to that of G. Moreover, it is known that there are nonisomorphic p-groups of Ulm type 2 having isomorphic Ulm sequences.

[1] Since completing this paper, I have learned that P. Hill and C. Megibben have obtained similar results.