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BOUNDARY VALUE PROBLEMS FOR A CLASS OF NONLINEAR DIFFERENTIAL EQUATIONS

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For certain functions f, positive in $(0, \infty)$ and continuous in $[0, \infty)$, the partial differential equation $\Delta x = x - xf(x^2)$ has spherically symmetric solutions $x_n(t)$, $n = 1, 2, \cdots$, which vanish at zero, infinity and $n-1$ distinct values in $(0, \infty)$. This and similar existence theorems for the ordinary differential equation $\ddot{y} - y + yF(y^2, t) = 0$ are proved by way of variational problems and the solutions are thus characterized by associated "eigenvalues". The asymptotic behavior of these eigenvalues is studied and some numerical data on the solutions is furnished for special cases of the above equations which are of interest in nuclear physics.

We begin by considering differential equations of the form

$$\ddot{y} - y + yF(y^2, t) = 0 , \tag{1.1}$$

were $F(\eta, t)$ satisfies the following conditions:

(Ia) $F(\eta, t)$ is continuous in η and t for $0 < t < \infty$ and $0 \leqq \eta < \infty$;

(Ib) $F(\eta, t) > 0$ for $\eta > 0$, $t > 0$;

(Ic) there exists a $\delta > 0$ such that, for every fixed positive t and $0 \leqq \eta_1 < \eta_2 < \infty$, $\eta_2^{-\delta} F(\eta_2, t) > \eta_1^{-\delta} F(\eta_1, t)$.

In the special case in which $F(y^2, t) = f(y^2/t^2)$, the substitution

$$x(t) = t^{-1} y(t) \tag{1.2}$$

transforms equation (1.1) into the form

$$\ddot{x} + \frac{2\dot{x}}{t} = x - xf(x^2) , \tag{1.3}$$

which is satisfied by spherically symmetric solutions of the partial differential equation

$$\Delta x = x - xf(x^2) , \tag{1.4}$$

where Δ is the three-dimensional Laplace operator and t denotes distance from the origin.

To simplify our statements concerning solutions of (1.1) and (1.3), we shall employ the following terminology.

DEFINITION I. A solution $y(t)$ of equation (1.1) which is continuous in $[0, \infty)$, positive in $(0, \infty)$, and satisfies $y(0) = 0$, $\lim_{t \to \infty} y(t) = 0$, shall be called a *fundamental solution of* (1.1) *for the interval* $[0, \infty)$.