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MULTIPLIERS AND H^* ALGEBRAS

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Let A be a normed algebra and $B(A)$ the algebra of all bounded linear operators from A into itself, with operator norm. An element $T \in B(A)$ is called a multiplier of A if $(Tx)y = x(Ty)$ for all $x, y \in A$. The set of all multipliers of A is denoted by $M(A)$. In the present paper, it is first shown that $M(A)$ is a maximal commutative subalgebra of $B(A)$ if and only if A is commutative. Next, $M(A)$ in case A is an H^*-algebra will be represented as the algebra of all complexvalued functions on certain discrete space. Finally, as an application of the representation theorem of $M(A)$, the set of all compact multipliers of compact H^*-algebras is characterized.

In case A is commutative, the general notion of multipliers was first studied by Helgason [7], followed by Wang [12] and Birtel [2], [3], [4]. In the special case when $A = L_1(G)$, the group algebra over an arbitrary locally compact abelian group, the problem of multipliers has also been studied by Helson [8] and Edwards [5]. (Cf. also Rudin [11].) Helgason [7] called a function g on the maximal ideal space $\mathscr{M}$ of A a multiplier if $g\hat{A} \subseteqq \hat{A}$ where $\hat{A}$ is the Gelfand transform of A. Later Wang [12] and Birtel [2] carried out more systematic studies on multipliers. In case A is semi-simple, Wang [12] proved that there exists a norm-decreasing isomorphism between $M(A)$ and $C^\infty(\mathscr{M})$, the algebra of bounded continuous functions of $\mathscr{M}$. In particular if $A = L_1(G)$, then $M(A) = M(G)$, the algebra of all bounded regular Borel measures on G. In the noncommutative case, Wendel [13] first studied multipliers[1] for noncommutative group algebras, followed by Kellogg [9] for H^*-algebras. However, since Kellogg's proofs rely heavily on the representation theorem of Wang [12] for multipliers on general commutative semi-simple Banach algebras, revelent results on multipliers of H^*-algebras were obtained only for the commutative case.

2. Multiplier algebras. Let A be a normed algebra. A is said to *without order* if either $xA = \{0\}$ or $Ax = \{0\}$ implies $x = 0$. Clearly, if A is semi-simple or A has a unit, then A is without order. In the sequal, we assume all normed algebras under consideration are without order. An element $T \in B(A)$ is called a *right* (*left*) multiplier

[1] Both Kellogg [9] and Wendel [13] used the terminology "centralizers" instead of "multipliers".