

ON THE CHARACTERIZATION OF MEASURES OF THE CONE DUAL TO A GENERALIZED CONVEXITY CONE

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We consider in this paper the cone $C(u_0, \dots, u_{n-1})$ of functions which are convex with respect to an Extended Complete Tchebycheffian system $\{u_0(t), u_1(t), \dots, u_{n-1}(t)\}$. The cone dual to $C(u_0, \dots, u_{n-1})$ is examined and necessary conditions as well as sufficient conditions for a measure to belong to this cone are developed. The merit of these conditions lies in the fact that they involve only the pattern of sign changes of the measure and related functions, and thus are easily verifiable.

Several applications are given. These include new inequalities for the Euler-Fourier coefficients of functions belonging to given convexity cones. Some new inequalities for the Fourier coefficients of the expansion of a function in a series of orthogonal polynomials are also obtained.

We consider in this paper the cone dual to a generalized convexity cone $C(u_0, \dots, u_{n-1})$ with respect to an Extended Complete Tchebycheffian system $\{u_0(t), u_1(t), \dots, u_{n-1}(t)\}$. The substantial role that these cones play in various areas of mathematics, such as moment theory, theory of approximation and interpolation and the theory of differential inequalities is discussed in detail in [5], (see also [4], [11], [6] and [7]). In a recent paper, Cargo [3] obtained independently for the special case when $n = 2$ and $u_0 \equiv 1$, some of the results of [4] and [11].

The dual cone was introduced by S. Karlin and A. Novikoff [4] who found necessary and sufficient conditions for a measure to belong to the dual cone. Applications of the results of [4] to the theory of reliability were later explored by Barlow and Marshall [1]. For the case $n = 2$ and $(u_0(t) \equiv 1, u_1(t) \equiv t)$ the conditions were stated earlier by Levin and Steckin [8], and a multidimensional version for this special case was recently obtained by Brunk [2].

The necessary and sufficient conditions involve some integral inequalities and thus are not always easily verifiable. Some simple sufficient conditions in terms of equalities and the pattern of sign changes of the measure under examination were also evolved in [4].

In this paper we intend to elaborate on this type of criteria, i.e., necessary conditions as well as sufficient conditions involving only equalities and the pattern of sign changes of the measure. As a by-product, we obtain the interesting fact that the dual cones are essentially mutually disjoint, e. g. no nontrivial measure can belong both to the dual