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INVARIANCE FOR LINEAR SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS

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In studying the existence and smoothness of invariant manifolds arising from nonlinear, perturbed systems of ordinary differential equations, one encounters the study of certain linear (in x), perturbation problems of the type

$$\dot{\theta} = a + \varepsilon b(\theta, \varepsilon)$$
$$\dot{x} = (A + \varepsilon B(\theta, \varepsilon))x$$

where θ and x are vectors, A and B are matrices, b and B are multiply periodic in θ, and ε is a perturbation parameter. Assuming A is a constant matrix consisting of square submatrices on the diagonal,

$$A = \operatorname{diag}(A_{11}, \cdots, A_{nn}),$$

with the maximum of the real parts of the eigenvalues of A_{jj} less than the minimum of the real parts of the eigenvalues of A_{kk} for $1 \leqq j < k \leqq n$; we construct a change of variables which reduces B to similar diagonal form.

For perturbed systems of nonlinear ordinary differential equations in a neighborhood of an invariant manifold, the existence and smoothness of the center-stable, center, and center-unstable manifolds is proved in § 6 of [3]. The method of proof used will also show the existence of other invariant manifolds, but for nonlinear systems the situation is not as simple as the associated linear problem with regard to finding invariant manifolds. R. Venti [7] has given linearization results for nonlinear systems of differential equations near a critical point. The results of this paper can be regarded as a first step in obtaining similar linearization results for nonlinear systems near an invariant manifold.

The techniques of this paper are based on those used by Y. Sibuya [5], [6]. Sibuya treats time-varying perturbation problems where the perturbation parameter enters in an analytic way. In § 3 of this paper we consider $C^k(1 \leqq k < \infty)$, θ-varying perturbation problems with θ representing the many-dimensional coordinates of some invariant manifold. In § 4 we give a counter-example to an analytic change of variables procedure, and then modify the procedure appropriately.

For linear systems of ordinary differential equations of the type

$$\dot{x} = (A + \varepsilon B(t, \varepsilon))x$$

(see (1) below with $\dim \theta = 1, \dot{\theta} = 1$, for details), where the matrix B