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ON THE ESSENTIAL SPECTRUM OF SCHROEDINGER OPERATORS WITH SINGULAR POTENTIALS

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In this paper, we show that under certain conditions the self-adjoint Schroediner operator $-\Delta_n + V(x)$ on $L_2(R_n)$, $n \geqq 1$, has essential spectrum $[0, \infty)$. The theorems improve previous results by permitting $V(x)$ to be more singular locally. The proof employs a factorization $V(x) = A(x)B(x)$ of the potential.

The essential spectrum of a self-adjoint operator is defined to consist of all points of the spectrum which are not isolated eigenvalues of finite multiplicity. Let $V(x)$ be a real-valued function on the n-dimensional Euclidian space R_n, and Δ_n the n-dimensional Laplacian. In a recent paper [6], Rejto gives conditions on V such that the operator $T = -\Delta_n + V(x)$ is self-adjoint with domain $\mathscr{D}(T) = \mathscr{D}(\Delta_n)$ and has essential spectrum $[0, \infty)$. His method consits essentially in proving compactness of the operator $VR_0(z)$, where $R_0(z) = (-\Delta_n - z)^{-1}$. The condition $\mathscr{D}(V) \supseteqq \mathscr{D}(\Delta_n)$, which accounts for the equality of the domains of T and Δ_n, is essential to this method, and corresponds very roughly to local square-integrability of $V(x)$.

Recent papers of Kato [3] and Kuroda [5] on the continuous spectrum and of Konno and Kuroda [4] on the discrete spectrum employ a method according to which one factors the potential into $V(x) = A(x)B(x)$ and considers the operator $AR_0(z)B$. In this theory, the operator T is defined by first defining its resolvent, and there is no guarantee that the domains of T and Δ_n are equal. This is rather an advantage, since it removes the requirement of local square-integrability. For example, Kato [3, §6] shows that if, for $n = 3$, the norm of V in $L_{3/2}(R_3)$ is sufficiently small, then T is unitarily equivalent to $-\Delta_3$.

In the present paper, we shall apply the factorization technique to the problem of invariance of the essential spectrum, extending Rejto's results to include potentials which are locally more singular. In particular, we remove certain seemingly artificial restrictions of [6] in the case of low ($n \leqq 3$) dimensions. For $n = 3$, our results will apply to $V(x) = |x|^{-a}$ for any $a < 2$.

For $n \geqq 3$, the definition and semi-boundedness of T are treated in §1, and the essential spectrum in §2. §3 is devoted to the proofs of the essential estimates for $n \geqq 3$. The special cases $n = 1$, 2 are discussed in §4.

For references to other work on essential spectra, we refer to the extensive bibliography of [6].