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A REMARK ON INTEGRAL FUNCTIONS OF SEVERAL COMPLEX VARIABLES

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Let R_ν, $\nu = \mathrm{I}, \mathrm{II}, \mathrm{III}, \mathrm{IV}$, be the 4 types of the classical Cartan domains and let $\mathscr{E}(R_\nu)$ denote the class of solutions u of the Laplace's equation $\Delta u = 0$ corresponding to the Bergman metric of R_ν which satisfy certain regularity conditions specified below.

In this note we give a distortion theorem for functions which are holomorphic in $\bar{R}_\nu$ and omit the value 0 there, and an application which leads to an interesting property of integral functions omitting the value 0. The tools used here are the generalized Harnack inequality for functions in the class $\mathscr{E}(R_\nu)$ and the classical theorem of Liouville for integral functions.

Let D be a bounded domain in the space C^p of p complex variables $z = (z^1, \cdots, z^p)$. The Laplace-Beltrami operator corresponding to the Bergman metric of D is

$$\Delta_D = T^{\alpha\bar{\beta}}\partial^2/\partial z^\alpha \partial \bar{z}^\beta \; ; \tag{1}$$

here $T^{\alpha\bar{\beta}}$ are the contravariant components of the metric tensor $T_{\alpha\bar{\beta}} = \partial^2 \log K_D/\partial z^\alpha \partial \bar{z}^\beta$ and $K_D = K_D(z, \bar{z})$ is the Bergman kernel function of D[1]. Let $\mathscr{E}(D)$ be the class of real functions u satisfying: (a) u is continuous in $\bar{D}$. (b) In $\bar{D} - \boldsymbol{b}(D)$, u is of C^2 and satisfies $\Delta_D u = 0$, where $\boldsymbol{b}(D)$ is the Bergman-Šilov boundary of D. It is well-known that the class $\mathscr{E}(D)$ solves the Dirichlet problems for certain types of bounded symmetric domains D ([3], [4]). These are the classical Cartan domains. Let z be a matrix of complex entries, z' its transpose, z^* its conjugate transpose and I the identity matrix. By $H > 0$ we mean that a hermitian matrix H is positive definite. The first 3 types are defined by $R_\nu = [z\colon I - zz^* > 0]$, $\nu = \mathrm{I}, \mathrm{II}, \mathrm{III}$, where z is an $m \times n$ matrix $(m \leqq n)$ for R_{I}, an $n \times n$ symmetric matrix for R_{II} and an $n \times n$ skew symmetric matrix for R_{III}. The fourth type R_{IV} is the set of all $1 \times n$ matrices satisfying the conditions:

$$1 + |zz'|^2 - 2zz^* > 0, \, |zz'| < 1 \, ,$$

or

$$1 > \bar{z}z' + [(\bar{z}z')^2 - |zz'|^2]^{1/2} \, .$$

By $\|z\|_\nu$ we denote the norm of the matrix $z \in R_\nu$, i.e., $\|z\|_\nu = \sup_{|x|=1}|zx|$, where x is an n-dimensional vector and $|x|$ the length