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GEOMETRIC THEORY OF A SINGLE MARKOV OPERATOR

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A Markov operator acting on the space of continuous functions on a compact Hausdorff space which is uniformly stable in the mean allows a topological ergodic decomposition. A partial converse to this is obtained; if the operator has a decomposition it is then uniformly stable in the mean when restricted to the conservative set. The characterization of uniformly mean stable operators in terms of its invariant structures is the major result. The problem of characterizing the manifolds which can be the invariant manifold for some Markov operator is also considered.

1. We denote by $C(X)$ the collection of all continuous real valued functions on the compact Hausdorff space X. A *Markov operator* on X is a bounded linear operator T taking $C(X)$ into $C(X)$ with $T1 = 1 = \|T\|$. A probability on X will always mean an element μ in the dual space $C(X)^*$ with $\mu(1) = 1 = \|\mu\|$; the w^*-compact convex set of all probabilities on X will be denoted by K. The following are immediate:

(a) K is invariant under T^*,

(b) T is order preserving on $C(X)$.

The Tychonov fixed point theorem gives

(c) there is a nonempty compact convex set K_F of T^*-invariant probabilities.

We let M be the closed linear manifold of invariant functions; then M contains the constant functions (and possibly nothing more). We obtain a natural decomposition of X from the equivalence relation $x \sim y$ if $f(x) = f(y)$ for all f in M. Each set D in this collection $\mathscr{D}$ of sets is closed for it is the intersection of level sets of continuous functions. A partition of a compact Hausdorff space is upper semicontinuous if it is the collection of level sets induced by a continuous map into another compact Hausdorff space [2, p. 132]. If we let Z be the product of the ranges of all of the invariant functions and define φ to be the map which sends x into the point of Z with $f(x)$ for its f^{th} coordinate we see that $\mathscr{D}$ is the level set partition of φ and so is upper semicontinuous. The quotient space $Y = X/\mathscr{D}$ is then compact Hausdorff and the projection map $\pi: X \to Y$ is continuous and closed. We can lift the map π to a map $\Pi: C(Y) \to C(X)$. The image set of Π consists of all continuous functions on X which are constant on each set of $\mathscr{D}$ and $\Pi^{-1}M$ is a subspace of $C(Y)$ which contains