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SIMULTANEOUS INTERPOLATION IN H_2, II

J. T. ROSENBAUM

Let $\{z_n\}$ denote a fixed sequence of complex numbers in the unit disc satisfying $(1 - |z_{n+1}|^2)/(1 - |z^n|^2) \leqq \delta < 1$ for some δ. Let M be a nonnegative integer, and let m be generic for integers between 0 and M inclusive. We define the linear functionals $L_n^{[m]}$ on H_2 by $L_n^{[m]}f = f^{(m)}(z_n)$. Given $M + 1$ sequences $w^{[0]}, \cdots, w^{[M]}$ in l_2, can there be found a function f in H_2 which solves the simultaneous weighted interpolation problem

$$f^{(m)}(z_n) = (w^{[m]})_n \, || L_n^{[m]} || \; ?$$

Shapiro and Shields considered this problem for $M = 0$. Their results were generalized by the author to the case $M = 1$. The purpose of this paper is to extend this generalization to arbitrary M.

The technique which we used for $M = 1$ would suggest that to proceed to arbitrary M, we should let $w^{[0]}, \cdots, w^{[M]}$ be prescribed in l_2 and then try to find $f_0, \cdots, f_M$ in H_2 satisfying

$$\text{(A)} \qquad \begin{cases} f_m^{(m)}(z_n) = (w^{[m]})_n \, || L_n{}^{[m]} || \\ f_m^{(i)}(z_n) = 0 \qquad (0 \leqq i \leqq M, \, i \neq m) \end{cases}$$

Then, $f_0 + \cdots + f_M$ could serve as the desired interpolating function. However, the computational difficulties which would be involved in such a program can be glimpsed even in the case $M = 1$. We found the following modification to be effective.

The work of Shapiro and Shields assures us that we can interpolate when $M = 0$. Fixing M and assuming the result for lesser values, let $w^{[0]}, \cdots, w^{[M]}$ be chosen from l_2. The induction hypothesis furnishes us with a function f_{M-1} corresponding to $w^{[0]}, \cdots, w^{[M-1]}$. We would like to alter f_{M-1} by finding a function g_{M-1} in H_2 for which the sum $f_M \equiv f_{M-1} + g_{M-1}$, together with its first M derivatives, assumes appropriate values on $\{z_n\}$. This is equivalent to demanding that

$$\begin{cases} g_{M-1}^{(M)}(z_n) = [(w^{[M]})_n - || L_n^{[M]} ||^{-1} f_{M-1}^{(M)}(z_n)] \, || L_n^{[M]} || \\ g_{M-1}^{(m)}(z_n) = 0 \qquad (m < M) \, . \end{cases}$$

By proving that the quantity in brackets is in l_2, we reduce the problem to that of finding a function g, once m and $w^{[m]}$ have been prescribed, which satisfies

$$\text{(B)} \qquad \begin{cases} g^{(m)}(z_n) = (w^{[m]})_n \, || L_n^{[m]} || \\ g^{(i)}(z_n) \; = 0 \qquad (i < m) \, . \end{cases}$$