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# FELLER BOUNDARY INDUCED BY A TRANSITION OPERATOR

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**A transition operator $T$ is a nonnegative contraction on an $AL$ space $L$ such that $\| T\mu \| = \| \mu \|$ for $\mu \geqq 0$. The set $\mathscr{M} = \{f \in L^*: T^*f = f\}$ of invariant functions of the adojoint $T^*$ turns out to be lattice isomorphic to $C(B)$ for a certain hyperstonian compact Hausdorff $B$. For the transition operator of a countable state Markov chain, $B$ is the Feller boundary of the process, and in the general case we call $B$ the Feller boundary induced by $T$. For the general case we exhibit several Markov processes associated with $T$ such that $B$ appears as a subset of the state space. These processes involve the potential theory of $T^*$. When $L$ is separable there is a quotient space $B_0$ of $B$ and a measure $\mu_0$ with $B_0$ as closed support such that $\mathscr{M}$ is isomorphic to $L_\infty(B_0, \mu_0)$. There is also a Markov process whose paths converge to $B_0$ with probability 1. However, we do not obtain the kernel representation of superharmonic functions as with the Martin-Doob boundary.**

2. The Feller boundary. Let $L$ be an $AL$ space (abstract $l$ space [3, Ch. VI]) and let $T$ be a bounded linear operator on $L$ with the properties $T \geqq 0$, $\| T\mu \| = \| \mu \|$ for $\mu \geqq 0$, so that $\| T \| = 1$. When $L$ is a space of measures our conditions are that $T$ transform probabilities into probabilities. As we will see, there are various discrete parameter Markov processes associated with $T$ such that $T$ induces the transition probabilities of the Markov processes, and we call $T$ a transition operator. Let $M$ be the Banach space conjugate of $L$, so that $M$ is an $AM$ space with order unit. The adjoint $T^*$ is an operator on $M$ such that $T^* \geqq 0$, $T^*1 = 1$, $\| T^* \| = 1$. Let $\mathscr{L}$ be the subspace of $L$ consisting of the invariant vectors $\mu = T\mu$ of $T$, and let $\mathscr{M}$ be the subspace of $M$ consisting of the invariant vectors $f = T^*f$ of $T^*$. We will see that $\mathscr{M}$ corresponds to the space of invariant (or harmonic or regular or concordant) functions of a Markov process, while $\mathscr{L}$ corresponds to a certain closed subspace of the invariant measures of the process. We will be concerned mainly with $\mathscr{M}$.

Let $X$ be the (Kakutani) space of lattice homomorphisms of $M$ onto the reals, with the topology induced by $M$, so that $M$ is isometrically linearly and lattice isomorphic to real $C(X)$ on hyperstonian compact Hausdorff $X$. From now on we identify $M$ with $C(X)$. We represent $M^*$ as the space rca $(X)$ of regular bounded signed Borel measures on $X$. For each $\nu \in \text{rca}(X)$, $\mathscr{S}(\nu)$ will denote the closed support of $\nu$. We denote by $\kappa: L \to \text{rca}(X)$ the natural injection of $L$ into its second