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ON (J, M, $\mathfrak{m}$)-EXTENSIONS OF ORDER SUMS OF DISTRIBUTIVE LATTICES

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In the first section of this paper a characterization of the order sum of a family $\{L_\alpha\}_{\alpha \in S}$ of distributive lattices is given which is analogous to the characterization of a free distributive lattice as one generated by an independent set. We then consider the collection Q of order sums obtained by taking different partial orderings on S. A natural partial ordering is defined on Q and its maximal and minimal elements are characterized.

Let J and M be collections of nonempty subsets of a distributive lattice L, and $\mathfrak{m}$ a cardinal. We define a $(J, M, \mathfrak{m})$-extension (ψ, E) of L, where E is a $\mathfrak{m}$-complete distributive lattice and $\psi: L \rightarrow E$ is a (J, M)-monomorphism. In the last section we define a $\mathfrak{m}$-order sum of a family of distributive lattices $\{L_\alpha\}_{\alpha \in S}$. The main result here is that the $\mathfrak{m}$-order sum exists if the order sum L of $\{L_\alpha\}_{\alpha \in S}$ has a $(J, M, \mathfrak{m})$-extension, where J and M are certain collections of subsets of L. These results are analogous to R. Sikorski's work in Boolean algebras (e.g., [6]).

1. Order sums. Let S be a fixed set and $\{L_\alpha\}_{\alpha \in S}$ a fixed collection of distributive lattices. From [2] it follows that for each poset $P = (S, \leqq)$, there exists a pair $(\{\varphi_\alpha\}_{\alpha \in S}, L(P))$, where $L(P)$ is a distributive lattice, and for each $\alpha \in S$, $\varphi_\alpha : L_\alpha \rightarrow L(P)$ is a monomorphism such that:

(1.1) L is generated by $\cup_{\alpha \in S} \varphi_\alpha(L_\alpha)$.

(1.2) If $\alpha < \beta$ then $\varphi_\alpha(x) < \varphi_\beta(y)$, for all $x \in L_\alpha$ and $y \in L_\beta$.

(1.3) If M is a distributive lattice and $\{f_\alpha : L_\alpha \rightarrow M\}_{\alpha \in S}$ is a family of homomorphisms such that $f_\alpha(x) \leqq f_\beta(y)$ whenever $\alpha < \beta$, $x \in L_\alpha$ and $y \in L_\beta$, then there exists a homomorphism f: $L(P) \rightarrow M$ such that $f\varphi_\alpha = f_\alpha$ for each $\alpha \in S$.

The pair $(\{\varphi_\alpha\}_{\alpha \in S}, L(P))$ will be called an *order sum of* $\{L_\alpha\}_{\alpha \in S}$ *over* P.

Let $\boldsymbol{P}$ be the family of all posets of the form $(S, \leqq)$ and let $Q = \{(\{\varphi_\alpha\}_{\alpha \in S}, L(P)) \mid P \in \boldsymbol{P}\}$. For $(\{\varphi_\alpha\}_{\alpha \in S}, L(P))$ and $(\{\theta_\alpha\}_{\alpha \in S}, L(P'))$ in Q we write

(1.4) $(\{\varphi_\alpha\}_{\alpha \in S}, L(P)) \leqq (\{\theta_\alpha\}_{\alpha \in S}, L(P'))$ provided:

(1.5) there is a homomorphism f: $L(P') \rightarrow L(P)$ such that $f\theta_\alpha = \varphi_\alpha$ for each $\alpha \in S$.

Note that (1.5) implies f is an epimorphism. If f is an isomor-