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AN EXTENSION OF HAIMO'S FORM OF HANKEL CONVOLUTIONS

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The real inversion formula for Hankel convolutions which is due to D. T. Haimo, is extended to certain generalized functions. That is accomplished by transferring the real inversion formula of D. T. Haimo onto the testing function space for the generalized function under consideration and then showing that the limiting process in the resulting formula converges with respect to the topology of the testing function space.

The Hirschman-Widder convolution transformation [3] has recently been extended to certain classes of generalized functions [7], [4] and their inversion formulae [3; pp. 127–132 and Theorems 7.1 b, p. 231] have been shown to be still valid when the limiting operation in those formulae is understood as weak convergence in the space D' of Schwartz distributions [5]. The purpose of the present work is to extend the inversion formula of D. T. Haimo for Hankel convolutions [1, p. 332] in a similar way to a certain space of generalized functions.

The notation and terminology of this work follows that of [7], [4]. I denotes the open interval $(0, \infty)$ and all testing functions herein are defined on I. Throughout this work x and y are variables over I unless otherwise mentioned. Finally, $D(I)$ is the space of smooth functions defined on I having compact supports. The topology of $D(I)$ is that which makes its dual the space $D'(I)$ of Schwartz distributions [5; Vol. I, p. 65] [8] on I. Thus, a sequence of functions $\{\phi_\nu\}_{\nu=1}^{\infty}$ is said to converge in $D(I)$ if and only if the supports of ϕ_ν are all contained within a fixed compact subset of I and for each k, $\{\phi_\nu^{(k)}\}_{\nu=1}^{\infty}$ converges uniformly on I.

2. The testing function space $\mathscr{G}(I)$. Let Δ_x stand for the operator $(D_x^2 + (2\gamma/x)D_x)$ where D_x is the differentiation operator and γ is a positive number. We say that a smooth function $\phi(x)$ defined over I belongs to $\mathscr{G}(I)$ if

$$(1) \qquad \gamma_k(\phi) = \sup_{0<x<\infty} | e^{cx}\Delta_x^{(k)}\phi(x) | < \infty$$

for all k assuming values $0, 1, 2, \cdots$. Here, c is a fixed real number; but in our later discussion we will choose c to be a fixed real number less than a_1. The topology in $\mathscr{G}(I)$ is generated by the collection of seminorms $\{\gamma_k\}_{k=0}^{\infty}$. Since γ_0 is a norm the collection of seminorms