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INCIDENCE MATRICES, INTERVAL GRAPHS AND SERIATION IN ARCHAEOLOGY

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The work of Fulkerson and Gross on incidence matrices shows that the question, whether a given incidence matrix A can be so re-arranged by rows as to bring together all the 1's in each separate column, can be settled if one merely knows A through the symmetrised product A^TA. Suppose it is known that such a row re-arrangement exists; it is proved here that A can then be re-arranged in the required way if one merely knows A through the dual symmetrised product, AA^T.

Thus A^TA and AA^T contain respectively (i) information sufficient to decide on the possibility or otherwise of such a re-arrangement, and (ii) information sufficient to determine a sorting algorithm.

Implications for archaeology are briefly discussed.

For the mathematical background to this paper the reader is referred to D. R. Fulkerson and O. A. Gross [1]; for archaeological motivations he is referred to the author's papers [3] and [4], and especially to the latter. There it is pointed out that the mathematical problems of recognizing incidence matrices with the 'consecutive 1's property' (i.e., zero-one matrices permitting a re-arrangement of rows which bunches the 1's in each separate column), and of sorting the rows of such a matrix so as to bring the 1's together, have much in common with the archaeological problem of 'sequence dating' first formulated in 1899 by Flinders Petrie. What distinguishes the two is that in the archaeological situation one is concerned with zero-one matrices permitting a re-arrangement of rows which only *approximately* bunches the 1's in each separate column. (Normally the rows represent say graves, and the columns represent objects or aspects of objects which may or may not be present in a given grave. The re-arrangement of rows determines the ordinal chronology of the graves, and this in turn assigns a range of 'sequence dates' to each object or feature). Despite this important difference, however, it is maintained here that the mathematical problems can be a convenient source of tentative heuristic algorithms for use in the archaeological one.

Note that incidence matrices having the consecutive 1's property are those which *after row re-arrangement* have a single bunch of 1's (if any) in each column. It is useful also to have a name for incidence matrices which display this pattern of 0's and 1's *as they stand*, no re-arrangement of rows being necessary; we shall therefore call such a matrix a *Petrie matrix*. Theorem 2.1 of [1] can then be formulated