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AUTOMORPHISMS OF GROUPS OF SIMILITUDES OVER F_3

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Let $Q(x)$ be a quadratic form defined on a vector space M of dimension $n = 2m > 4$ ($n \neq 8$) over the field with three elements. The purpose of this paper is to show that any automorphism of the projective group of similitudes or of the projective group of proper similitudes can not take the coset of an $(n-2, 2)$ involution into the coset of an $(n-p, p)$ involution or into the coset of a similitude T of ratio ρ where $T^2 = \rho_L$ (left multiplication by ρ) and where ρ is not a square in F_3. This result, together with some results of Wonenburger, shows that any such automorphism is induced by an automorphism of the group of similitudes.

In [6] Wonenburger proved the above statements except when the field K had 3 or 5 elements. Only the corollary to Lemma 3 of [6] needs to be verified to complete this result for all fields of characteristic not 2. The case in which $K = F_5$ can be proved by a slight alteration of Wonenburger's argument, so it will not be considered in this paper.

A linear transformation T of a vector space M of dimension $n = 2m$ is called a similitude of ratio ρ ($0 \neq \rho \in K$), of the quadratic form Q, if for every $x \in M$, $Q(xT) = \rho Q(x)$. The similitudes of ratio 1 make up the orthogonal group $O(Q)$. A similitude T of ratio ρ is called a proper similitude if the determinant of the matrix of T, Det $T = \rho^m$, and improper if Det $T = -\rho^m$. Let $S(Q)$ ($S^+(Q)$) denote the group of similitudes (proper similitudes). In the projective groups $PS(Q) = S(Q)/K^*$ and $PS^+(Q) = S^+(Q)/K^*$ we will use the symbol $\bar{T}$ to denote the coset of the similitude T.

A projective involution is a similitude T such that $\bar{T}^2 = \bar{1}$, the coset of the identity. There are three types of projective involutions (see [6, p. 608]). First there are scalar multiples of orthogonal involutions. With respect to an orthogonal involution T, we can decompose M into a "minus space" M^- of dimension p such that $xT = -x$ for all $x \in M^-$, and a "plus space" M^+ of dimension $n-p$ such that $xT = x$ for all $x \in M^+$. Such a transformation will be called an $(n-p, p)$ involution and its image in $PS(Q)$ or in $PS^+(Q)$ will be called an $(n-p, p)$ coset. A P-involution is a similitude T of ratio ρ with $T^2 = -\rho_L$, left multiplication by $-\rho$. All P-involutions are proper similitudes (see [3, p. 64]). A P'-involution is a similitude T of ratio ρ such that $T^2 = \rho_L$, ρ not a square in K. If $K = F_3$, it can be easi-