PACIFIC JOURNAL OF MATHEMATICS
Vol. 29, No. 1, 1969

INDEFINABILITY IN THE ARITHMETIC ISOLIC INTEGERS

PHILIP OLIN

This paper is primarily concerned with the theory of the arithmetic isolic integers. The following results are obtained:

(1) No nonfinite member of the arithmetic isolic integers $\Lambda^*(A)$ can be defined in $\Lambda^*(A)$ even by an infinite number of arithmetic formulas (Theorem 4).

(2) The arithmetic isols $\Lambda(A)$ cannot be defined in $\Lambda^*(A)$ even by an infinite number of arithmetic formulas (Theorem 7).

(3) We exhibit some nonstandard models of arithmetic contained within $\Lambda^*(A)$ (Theorem 10).

The first result above follows from recent work of Nerode in the theory of isols, while the second strengthens his results to obtain the desired conclusion.

The ring of arithmetic isolic integers $\Lambda^*(A)$ was introduced by Nerode in [5] where he showed that $\Lambda^*(A)$ is elementarily equivalent to a reduced power Q of the ring of integers and where he adapted the method of Feferman and Vaught [1] to $\Lambda^*(A)$. In [6] Nerode gave a procedure for finding isols and isolic integers which satisfy extensions to isols of recursively enumerable relations. The work which follows here both uses these results and, in some cases, strengthens them. We use the definitions and notation of [4], [5], and [6].

It is possible that a nice structure theorem for $\Lambda^*(A)$ can be proved; Nerode has asked whether $\Lambda^*(A)$ is isomorphic to Q, the reduced power of the integers just mentioned (see remarks following Corollary 5 below). E. Ellentuck has obtained such a result for the ring of Dedekind finite cardinals in the models of a particular extension of Zermelo Fraenkel set theory without the axiom of choice. However even if such a result is obtained for $\Lambda^*(A)$, since $\Lambda(A)$ cannot be defined in $\Lambda^*(A)$ by an infinite number of arithmetic formulas, it will still not be possible to arithmetically define in this way the substructure corresponding to $\Lambda(A)$.

1. We adopt the notation of [4], [5], and [6]. As in [5] and [7], $Q = Z^\omega/D_0$ where Z is the ring of integers and D_0 the filter of cofinite subsets of ω.

THEOREM 1. *Suppose $\{\varphi_i\}_{i<\omega}$ is a collection of arithmetic formulas in the variables $\bar{v} = (v_0, v_1, v_2, \cdots)$ and the collection is finitely satisfiable in Q. Then there exist $A, B \in Q^\omega$ such that*