PACIFIC JOURNAL OF MATHEMATICS
Vol. 29, No. 1, 1969

SOME INEQUALITIES FOR STARSHAPED AND CONVEX FUNCTIONS

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Necessary and sufficient conditions are obtained on a function G of bounded variation such that $\phi\left(\int x(t)dG(t)\right) \leqq \int \phi(x(t))dG(t)$ for all increasing x for which $x(t_0) = 0$ for some specified t_0, and all convex ϕ for which $\phi(0) = 0$; the conditions are otherwise independent of ϕ and x. Similar results are obtained when the inequality is reversed. Necessary and sufficient conditions for both directions of inequality are also obtained when ϕ is starshaped and $\phi(0) = 0$.

The relationship to previous results is sketched. Applications to statistical tolerance limits are indicated.

Several inequalities are known that give necessary and sufficient conditions for a signed measure μ to satisfy

$$\int \phi(x)d\mu(x) \geqq 0 \tag{1.1}$$

for all functions ϕ in a given convex cone. For example, such results were obtained by Hardy, Littlewood and Pólya [7] for the cone of convex functions, and by Karlin and Novikoff [9], Ziegler [17] and Karlin and Studden [10] for cones of generalized convex functions.

By changing variables in such a result, it is easy to obtain conditions on μ in order that

$$\int \phi(x(t))d\mu(t) \geqq 0 \tag{1.2}$$

for all ϕ in the given convex cone, where x is an increasing function. Generally speaking the conditions so obtained depend upon the function x. In some applications, x is replaced by a random function (see Barlow and Proschan [1]). Inequalities are thus required which will hold for essentially all possible realizations of the random function, so that those obtained via a change of variables, like (1.2), are not useful.

In this paper, we consider only measures μ which are the difference between a measure ν and the measure which has unit mass concentrated *at the point* $\int x(t)d\nu(t)$. *Consequently, all the inequalities that we* obtain have either the form

$$\phi\left(\int x(t)d\nu(t)\right) \leqq \int \phi(x(t))d\nu(t) \tag{1.3}$$