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UNCONDITIONAL AND SHRINKING BASES IN LOCALLY CONVEX SPACES

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Let E be a locally convex space with an unconditional Schauder basis $\{x_k\}$ and let $\{f_k\}$ be the sequence of coefficient functionals biorthogonal to $\{x_k\}$. Owing to works of R. C. James and S. Karlin it is known that if E is a Banach space then each of the three conditions which follow is necessary and sufficient for $\{f_k\}$ to be a basis for E^* in the strong (norm) topology.

(1) E has no subspace topologically isomorphic to the space l^1.

(2) E^* is separable in the strong topology.

(3) E^* is weakly $(w(E^*, E^{**}))$ sequentially complete.

The primary purpose of this paper is to show that in certain spaces which are more general than Frechet spaces and hence than Banach spaces, each of the above three conditions is necessary and sufficient for

(0) $\{f_k\}$ is a strong basis for E^*.

Specifically if E is a complete barrelled space, each of (1) and (2) is sufficient for (0). In any locally convex space (2) implies (1) (even if E has no basis) and so each of (1) and (2) is necessary for (0). If E is a space having the property that weak* bounded subsets of E^* are strongly bounded (complete locally convex spaces and barrelled spaces have this property) then (3) is sufficient for (0). (3) is necessary for (0) if E is both barrelled and metrizable.

Besides the papers of James [8] and Karlin [9], related matter of importance is contained in the works of M. M. Day [2, Ch. 4] and C. Bessaga and A. Pelczynski [1]. E. L. Dubinsky and J. R. Retherford [5] using Köthe sequence space techniques have proved the (1) $\leftrightarrow$ (2) part of Theorem 2.12.

1. Preliminarie and fundamental theorems. Since the main results of this paper depend upon many theorems which are not widely known in their more general settings, the author thought it wise to include this section which introduces, in addition to basic theorems, some difinitions, terminology, and notation. No proofs are given for known results.

If E is a locally convex space, then π will be used for the canonical mapping from E to E^{**}, the space of strongly continuous functions on E^*. For convenience, a subscript 2 will be added to the word "space" to designate that the Hausdorff axiom is satisfied.