PACIFIC JOURNAL OF MATHEMATICS
Vol. 29, No. 3, 1969

POWER SERIES RINGS OVER A KRULL DOMAIN

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Let D be a Krull domain and let $\{X_\lambda\}_{\lambda \in \Lambda}$ be a set of indeterminates over D. This paper shows that each of three "rings of formal power series in $\{X_\lambda\}$ over D" are also Krull domains; also, some relations between the structure of the set of minimal prime ideals of D and the set of minimal prime ideals of these rings of formal power series are established.

In considering formal power series in the X_λ's over D, there are three rings which arise in the literature and which are of importance. We denote these here by $D[[\{x_\lambda\}]]_1$, $D[[\{X_\lambda\}]]_2$, and $D[[\{X_\lambda\}]]_3$. $D[[\{X_\lambda\}]]_1$ arises in a way analogous to that of $D[\{X_\lambda\}]$—namely, $D[[\{X_\lambda\}]]$ is defined to be $\bigcup_{F \in \mathscr{F}} D[[F]]$, where $\mathscr{F}$ is the family of all finite nonempty subsets of Λ. $D[[\{X_\lambda\}]]_2$ is defined to be

$$\left\{\sum_{i=0}^{\infty} f_i \mid f_i \in D[\{X_\lambda\}],\ f_i = 0 \text{ or a form of degree } i\right\},$$

where equality, addition, and multiplication are defined on $D[[\{x_\lambda]]_2$ in the obvious ways. $D[[\{X_\lambda\}]]_2$ arises as the completion of $D[\{X_\lambda\}]$ under the $(\{X_\lambda\})$-adic topology; the topology on $D[[\{X_\lambda]]_2$ is induced by the decreasing sequence $\{A_i\}_0^\infty$ of ideals, where A_i consists of those formal power series of order $\geqq i$—that is, those of the form $\sum_{j=i}^{\infty} f_j$. If Λ is infinite, A_1 properly contains the ideal of $D[[\{X_\lambda\}]]_2$ generated by $\{X_\lambda\}$. Finally, $D[[\{X_\lambda\}]]_3$ is the *full* ring of formal power series over D, and is defined as follows (cf. [1, p. 66]): Let N be the set of nonnegative integers, considered as an additive abelian semigroup, and let S be the weak direct sum of N with itself $|\Lambda|$ times. S is an additive abelian semigroup with the property that for any $s \in S$, there are only finitely many pairs (t, u) of elements of S whose sum is s. $D[[\{X_\lambda\}]]_3$ is defined to be the set of all functions $f: S \to$ D, where $(f+g)(s) = f(s) + g(s)$ and where $(fg)(s) = \sum_{t+u=s} f(t)g(u)$ for any $s \in S$, the notation $\sum_{t+u=s}$ indicating that the sum is taken over all ordered pairs (t, u) of elements of S with sum s. To within isomorphism we have $D[[\{X_\lambda\}]]_1 \subseteqq D[[\{X_\lambda\}]]_2 \subseteq D[[\{X_\lambda\}]]_3$, and each of these containments is proper if and only if Λ is infinite. Our method of attack in showing that $D[[\{X_\lambda\}]]_i$, $i = 1, 2, 3$, is a Krull domain if D is consists in showing that $D[[\{X_\lambda\}]]_3$ is a Krull domain and that $D[[\{X_\lambda\}]]_3 \cap K_i = D[[\{X_\lambda\}]]_i$ for $i = 1, 2$, where K_i denotes the quotient field of $D[[\{X_\lambda\}]]_i$.

1. **The proof that $D[[\{X_\lambda\}]]_3$ is a Krull domain.** Using the