PACIFIC JOURNAL OF MATHEMATICS
Vol. 30, No. 1, 1969

REPRESENTATIONS OF PRIMARY ARGUESIAN LATTICES

BJARNI JÓNSSON AND GEORGE S. MONK

The principal result of this paper states that every primary Arguesian lattice of geometric dimension three or more is isomorphic to the lattice of all submodules of a finitely generated module over a completely primary uniserial ring.

We shall discuss here briefly the background of this problem. Insofar as it is needed in this paper, the terminology employed here will be defined later.

In view of the correspondence between projective geometries and complemented modular lattices (cf. [4], p. 93) it follows from the classical coordinatization theorem for projective geometries (cf. [5], Chap. VI), that for $n \geqq 4$ the simple, complemented modular lattices of dimension n coincide up to isomorphism with the lattices of all subspaces of n-dimensional vector spaces over division rings or, equivalently, with the lattices of all left ideas of full rings of n by n matrices over division rings. This result has been generalized in two directions. On the one hand, von Neumann showed in [10] that every complemented modular lattice which has a homogeneous n-frame, $n \geqq 4$, is isomorphic to the lattice of all principal left ideals of a regular ring. On the other hand, Baer proved in [3] that every primary lattice of geometric dimension $n \geqq 6$ is isomorphic to the lattice of all submodules of a finitely generated module over a completely primary uniserial ring. Conversely, the lattice of all submodules of a finitely generated module over a completely primary uniserial ring is always a primary lattice. To indicate why these results are interesting, we remark that the class of rings involved contains the ring of integers modulo a prime power, and the corresponding class of modules therefore contains all finite primary Abelian groups. Baer's result was rediscovered by Inaba, who in [6] gave a different proof and, more important, replaced the condition $n \geqq 6$ by $n \geqq 4$.

Even for finite dimensional complemented modular lattices these results cannot be extended to the case $n=3$ because of the existence of projective planes in which Desargues' Law fails. In [11] Shützenberger observed that a projective geometry satisfies Desargues' Law if and only if a certain identity holds in the corresponding lattice. We adopt here a variant of Schützenberger's identity that was introduced in [8], and call a lattice Arguesian in case it satisfies this identity. Thus, for $n \geqq 3$ the simple, complemented Arguesian lattices of dimension n coincide up to isomorphism with the lattices of all